Gravitation

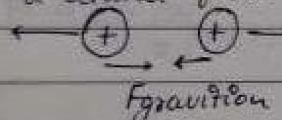
GRAVITATION

Mass

- Cause of gravitational force
- Property of body due to which it apply and experience Gravitational force

Gravitational force

- Every matter in this universe attracts every other matter by a force is called gravitional force
- It is always attractive $\quad F_g << F_{electrostatic}$
- It is weakest force
- It is a central force

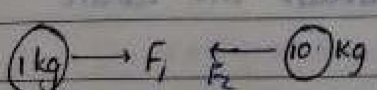


- " Every particle of matter in the universe attracts every other particle with a force that is directly proportional to the product of the masses of the particles and inversely proportional to the square of the distance b/w them. "

$$|\vec{F_{21}}| = |\vec{F_{12}}| \propto m_1 m_2$$
$$\propto \frac{1}{r^2}$$
$$|\vec{F_{21}}| = \frac{m_1 m_2}{r^2} = \frac{G m_1 m_2}{r^2}$$

('G' is universal gravitational constant) $G = 6.67 \times 10^{-11}\ N\,m^2/kg^2$

$$F_{12} = F_{21} = \frac{G m_1 m_2}{r^2} \rightarrow \text{Scalar form}$$



F_1 = Force on 1kg due to 10 kg

F_2 = Force on 10kg due to 1kg

$F_1 = \frac{G(1 \times 10)}{r^2} \quad F_2 = \frac{(10 \times 1)G}{r^2}$

$\therefore F_1 = F_2$

$$\boxed{\vec{F_{21}} = \frac{-Gm_1m_2}{|\vec{r}_{21}|^3}\,\vec{r}_{21}} \rightarrow \text{Vector form of gravitational force}$$

or $$\vec{F_{21}} = \frac{-Gm_1m_2}{|r_{21}|^2}\,\hat{r}_{21}$$

NEWTON'S LAW OF GRAVITATION (UNIVERSAL LAW OF GRAVITATION)

- Only valid for point mass and spherical symmetrical, (ring, rod) etc. → not valid.
- Can't be proved but can verify only.
- Always attractive.
- Weakest force in nature
- follow inverse square law.
- long range.
- central (conservative)
- does not depend on medium. because
- ~~medium~~ mediated by ~~gravitation~~ graviton
- reason of stability of universe is gravitational force
- form action reaction pair.
- Controls the rotational motion of satellites and planets.

Q Gravitation is the phenomenon of interaction between.
a) Point masses only.
b) Any arbitrary shaped mass.
c) Planets only
d) Point mass and spherical mass symmetry.

Q The gravitational force with which the earth attracts the moon
→ is equal to the force with which moon attracts the earth

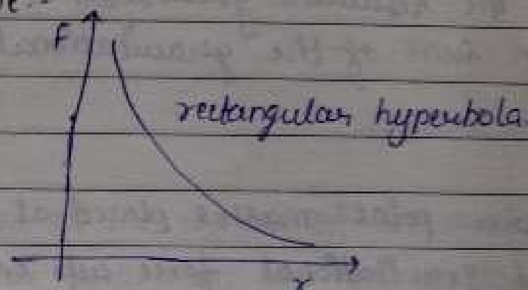
Q The dimensional formula of 'G' is
→ $M^{-1}L^{3}T^{-2}$

Q Force of gravitation b/w two masses is found to be F in vacuum. If both the masses are dipped in water at same distance then, new force will be

→ F

If somehow the distn b/w the sun and the earth is doubled, the gravitational force b/w them will become:-

→ one fourth



Q If the distance b/w them becomes double and their mass reduced to half then Gravitational force becomes

Ans $F = G\frac{\frac{m_1}{2}\frac{m_2}{2}}{(4R^2)} = \frac{Gm_1m_2}{16R^2} = \frac{1}{16}$ times.

Q If distance b/w two objects increases by 25% then % change in force

Ans $F = \frac{Gm_1m_2}{r^2}$ → constant

$r' = 125\% \, r = \frac{125r}{100} = \frac{5}{4}r$

$F = \frac{1}{r^2} \rightarrow \frac{1}{(\frac{5}{4})r} = \frac{1}{\frac{25r^2}{16}} = \frac{16}{25} \times 100 = 64\%$

∴ % change = −36% or 36% decrease

Q If distance b/w two object decrease by 50% then % change in Force

Ans $F = \frac{1}{r^2} = \quad r' = 50\% \, r = \frac{50}{100} = \frac{1}{2}r$

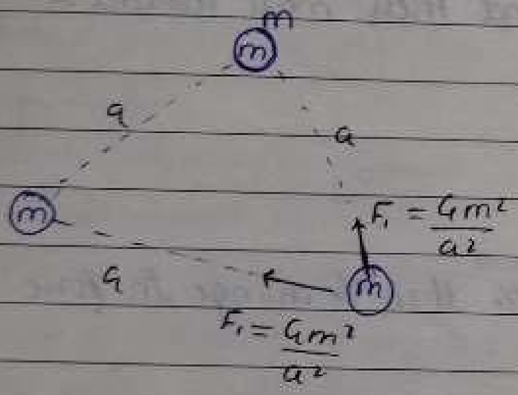
$F = \frac{1}{\frac{1}{4}r^2} = 4 \times 100 = 400\% - 100 = 300\%$

LIMITED EDITION

Principle of Superposition of Gravitional Forces

If we have a collection of point masses, the gravitational forces between any two point masses acts independently an uninfluenced by the presence of other masses.
Hence, the resultant gravitational force acting on any one of them is the vector sum of the gravitational forces exerted by the other point masses.

Q Three point masses placed at the corner of equilateral triangle then find gravitational force acts on one due to other 2-particle

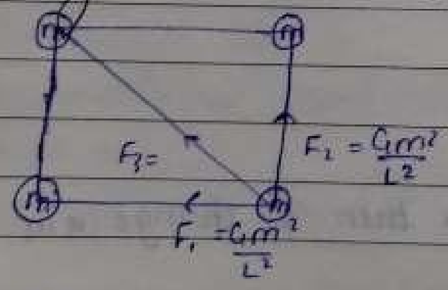


MR* vector

$F_{net} = \sqrt{3}\,F = \frac{\sqrt{3}\,Gm_1m_2}{r^2} = \frac{\sqrt{3}\,Gm^2}{a^2}$

Q Four identical point mass placed at the corner of square then net force on any one due to other 3-particle

Ans



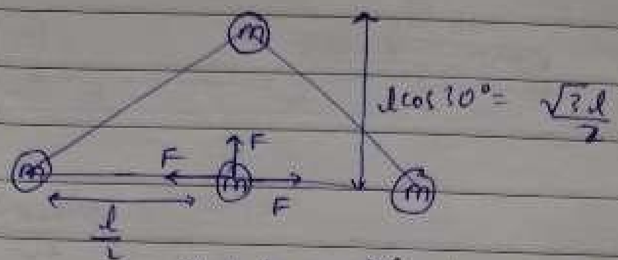
$F_3 = \frac{Gm^2}{2L^2}$

$\vec{F_1} + \vec{F_2} = \frac{\sqrt{2}\,Gm^2}{L^2}$

$\vec{F_1} + \vec{F_3} = \frac{Gm^2}{2L^2} + \frac{\sqrt{2}\,Gm^2}{L^2} = \frac{Gm^2 + 2\sqrt{2}\,Gm^2}{2L^2} = \frac{(1+2\sqrt{2})\,Gm^2}{2L^2} = \left(\frac{1}{2}+\sqrt{2}\right)\frac{Gm^2}{L^2}$

Q Three particles each of mass m are kept at the corners of an equilateral triangle of side 'l'. Force exerted by this system on another particle of mass m placed at the midpoint of any side will be:

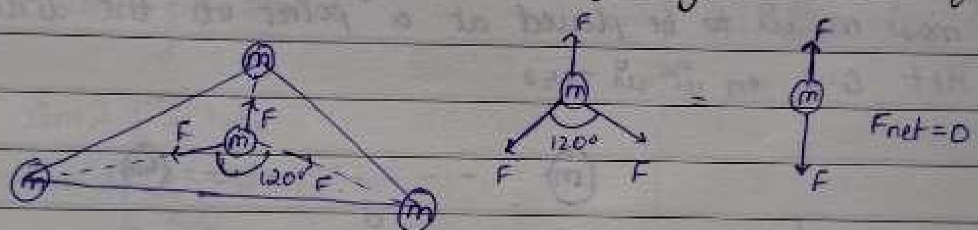
Ans



Net force $= \frac{4Gm^2}{3l^2} = \frac{4Gm^2}{3l^2}$

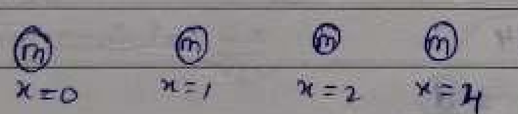
Q Three particles each of mass m are kept at the corners of an equilateral triangle of side l. Force exerted by this system on another particle of mass m placed at the ~~midpoint of any point~~ centre of the triangle

Ans



Q A large number of identical point masses m are placed along x axis, at $x = 0, 1, 2, 4 \ldots$ The mag of gravitational force on mass at origin $(x=0)$ will be

Ans



$$F = Gm^2 + \frac{Gm^2}{4} + \frac{Gm^2}{16} + \frac{Gm^2}{64} \ldots$$

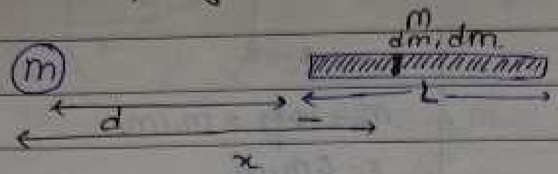
$$F = Gm^2\left(1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} \ldots\right)$$

Sum of G.P. series $= \frac{a}{1-r} = \frac{1}{1-\frac{1}{4}} = \frac{4}{3}$

$$F = \frac{4Gm^2}{3}$$

Q A particle of mass m is placed at a distance d from one end of a uniform rod with length L and mass M as shown in the figure. Find the mgn of the G.F. on the particle due to the rod.

Ans



$dF = \frac{Gm\,dm}{x^2}$

$\lambda = \frac{dm}{dx}$ $\quad dm = \lambda\,dx$

as uniform rod $\therefore \lambda = \frac{M}{L}$

$\int dF = \int \frac{Gm\lambda\,dx}{x^2}$

$F = Gm\lambda \int_d^{d+L} \frac{1}{x^2}\,dx$

$F = Gm\lambda \int_d^{L+d} x^{-2}\,dx$

$F = Gm\lambda \left(-\frac{1}{x}\right)_d^{L+d}$

$F = -Gm\lambda \left(\frac{1}{x}\right)_d^{L+d}$

$F = -Gm\lambda \left(\frac{1}{L+d} - \frac{1}{d}\right)$

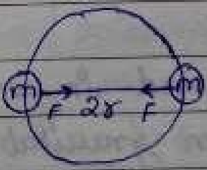
$F = -Gm\lambda \left(\frac{\cancel{d} - L - \cancel{d}}{(L+d)d}\right)$

$F = +\frac{Gm\lambda L}{(L+d)d} = \frac{Gm\frac{M}{\cancel{L}}\times\cancel{L}}{(L+d)d} = \frac{GmM}{(L+d)d}$

$\therefore$ Ans $F = \frac{GmM}{(L+d)d}$ Ans

Q Two particles of equal mass m are moving round a circle of radius r due to their mutual gravitational interaction. Find the time period of each particle.

Ans Time period $= \frac{2\pi}{\omega}$



$F = \frac{Gm^2}{4r^2}$

$a_c = \frac{Gm}{4r^2}$

$T^2 \propto r^3$

$a_c = \omega^2 R \Rightarrow \frac{Gm}{4r^2} = \omega^2 R$

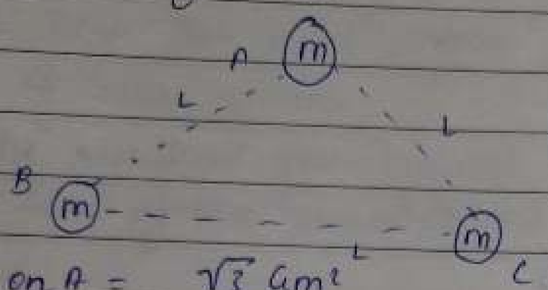
$\frac{\sqrt{Gm}}{2r\sqrt{r}} = \omega$

$T = \frac{2\pi}{\omega} = \frac{4\pi r\sqrt{r}}{\sqrt{Gm}}$ T Ans

LIMITED EDITION

Q Three particles A, B, C each of mass m are lying at the corners of an equilateral triangle of side L. If the particle A is released keeping the particle B and C fixed, the magn of instantaneous acceleration of A is.

Ans

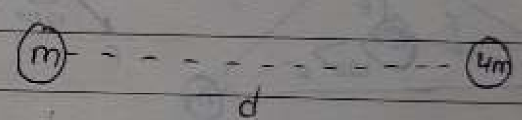


F_{net} on A $= \dfrac{\sqrt{3}\, Gm^2}{L^2}$

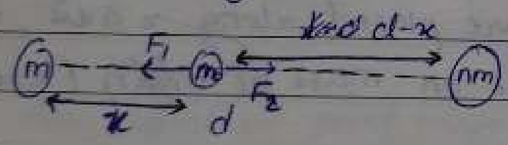
$a = \dfrac{\sqrt{3}\, Gm^2}{L^2 \times m} = \dfrac{\sqrt{3}\, Gm}{L^2}$

Q Two point masses m and 4m are separated by a distn d on a line. A third point mass m_0 is to be placed at a point on the line such that the net G.F. on it is zero

Ans



Creation of MR*



$F_1 = F_2$

$\Rightarrow \dfrac{Gm m_0}{x^2} = \dfrac{Gm m_0 n}{(d-x)^2}$

$n x^2 = (d-x)^2$

$\sqrt{n}\, x = d - x$

$\sqrt{n}\, x + x = d$

$x(\sqrt{n}+1) = d$

$\boxed{x = \dfrac{d}{\sqrt{n}+1}}$

If $n = 1$

then $x = \dfrac{d}{2}$

Answer for this question =

$x = \dfrac{d}{\sqrt{n}+1}$

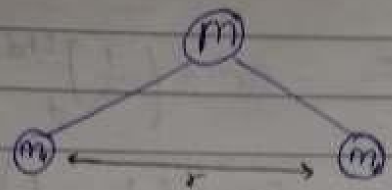
$n = 4$

$x = \dfrac{d}{2+1} = \dfrac{d}{3}$

LIMITED EDITION

Q A mass M broke into two parts of masses m_1 and m_2. How are m_1 and m_2 related so that force of attraction towards between the two parts is maxm ?

Ans



$$F = \frac{G m_1 m_2}{r^2} \qquad m_1 + m_2 = M$$
$$m_2 = M - m_1$$

F_{max} = When $\frac{dF}{dm_1} = 0$
↳ because r is constant

$$F = \frac{G m_1 (M - m_1)}{r^2}$$

$$\frac{dF}{dm_1} = 0 = \frac{G}{r^2} \frac{d}{dm_1} \, m_1 M - m_1^2$$

$$0 = \frac{G}{r^2} \, M - 2m_1$$

$$\boxed{M = 2m_1}$$

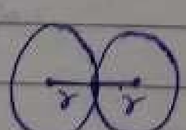
$$m_1 = \frac{M}{2}$$
$$m_2 = \frac{M}{2}$$
$$\therefore \boxed{M = 2m}$$

MR*
$M = 10kg = m_1 + m_2$
$F = \frac{G m_1 m_2}{(m_1+m) r^2}$

$8 \times 2 = 16$
$9 \times 1 = 9$
$7 \times 3 = 21$
$6 \times 4 = 24$
$5 \times 5 = 25$

Q Two identical sphere are placed in contact with each other. If r is the radius of each sphere, then gravitational att^n b/w the two is proportional to.

Ans $F = \frac{G m_1 m_2}{r^2}$ =



$= \frac{G m^2}{4r^2}$

To find relation b/w F and G, ~~and~~ m^2 should be constant but m is not constant, it is dependable on radius but at the same time we can find that for identical object density is constant

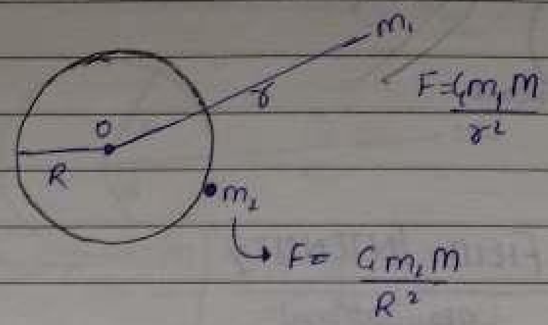
$$\therefore F = \frac{G(\rho V)^2}{4r^2} = \frac{G(\rho \frac{4}{3}\pi)^2 r^6}{r^2} = G(\rho \tfrac{4}{3}\pi)^2 r^4$$

$\therefore F \propto r^4$ Ans

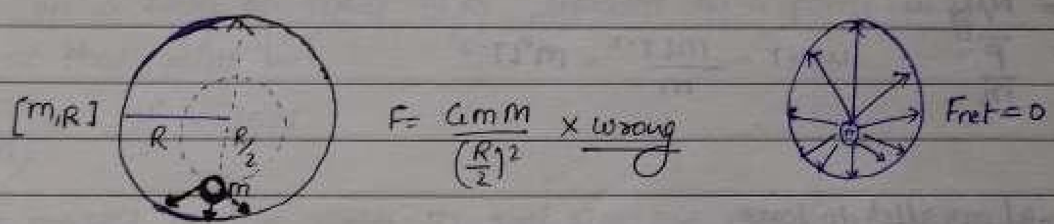
$F \propto r^4$

LIMITED EDITION

Q Find G.F. b/w hollow sphere & point mass



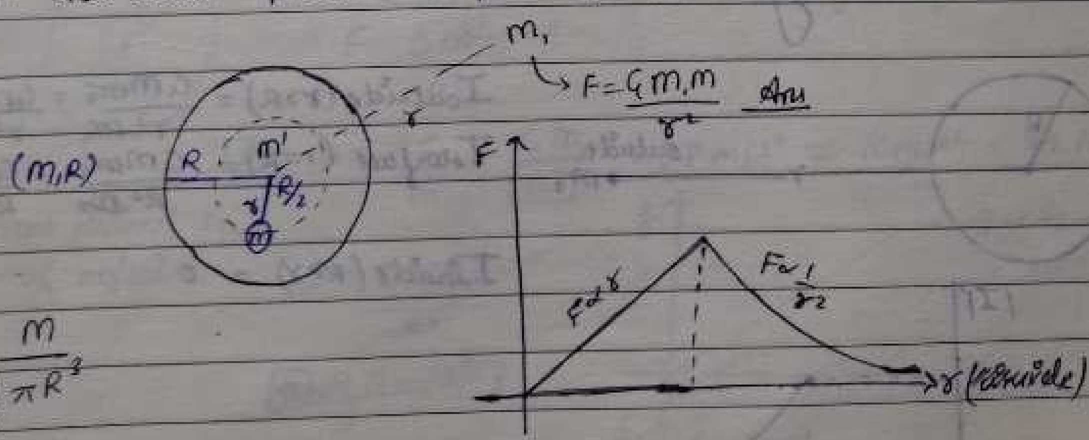
Q Find G.F. b/w Hollow sphere and point mass ??



इस question के लिए Newton ने Shell theorem for law of Gravitation दिया था जिसके ~~का~~ हमें एक sphere assume करते हैं जिसका circumference mass m से पास हो रहा हो और जितना force of attraction ~~उस~~ वह ~~इस~~ imaginary sphere लगाएगा उतना ही उस mass m पर force लग रहा होगा ~~उस~~ क्योंकि इस question में imaginary sphere का mass '0' hai इसलिए mass m में लगाने वाला force '0' hoga

$\therefore$ Ans = 0

Q Find G.F. b/w solid sphere and point ~~sphere~~ mass.



$\rho = \frac{M}{\frac{4}{3}\pi R^3}$

$m' = \rho V'$

$m' = \frac{M}{\cancel{\frac{4}{3}\pi} R^3} \cancel{\frac{4}{3}\pi} r^3 = \frac{M}{R^3} r^3$ $\qquad$ Force $= \frac{GmMr^{\cancel{3}}}{R^3 \times \cancel{r^2}} = \frac{GmMr}{R^3}$

Solid के बाहर force $F \propto \frac{1}{r^2}$

Solid के अंदर Force $F \propto r$ $\left(\frac{GMmr}{R^3}\right)$ ↳ constant

GRAVITATIONAL FIELD INTENSITY

Gravitational force per unit mass is called Gravitational field intensity

UNIT = N/kg

$\vec{I} = \frac{\vec{F}}{m}$ UNIT $= \frac{MLT^{-2}}{M} = M^0LT^{-2}$

Vector

Directional parallel to force

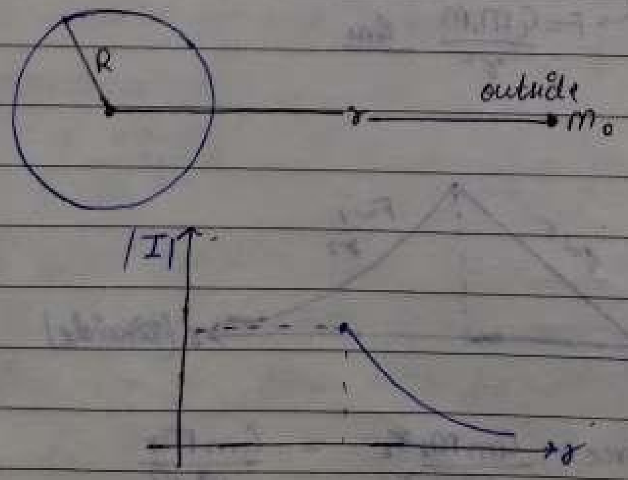
(i) Gravitational field intensity due to point mass.

$F = \frac{Gmm_0}{r^2}$

$I = \frac{F}{m_0} = \frac{Gm}{r^2}$

$\vec{I} = -\frac{Gm}{r^2}\hat{r}$

(ii) Gravitational field Intensity due to hollow sphere

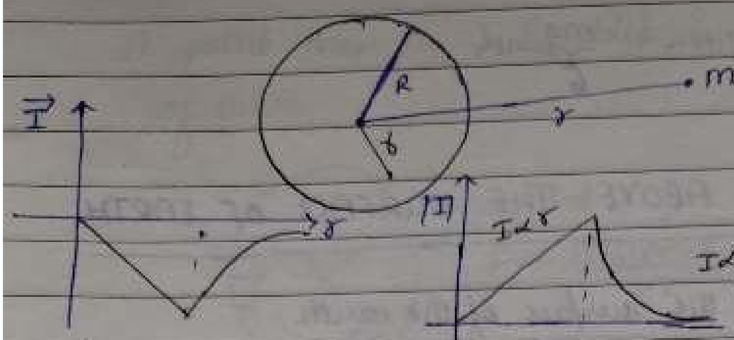


$I_{outside} (r>R) = \frac{GMm_0}{r^2 m_0} = \frac{GM}{r^2}$

$I_{surface} (r=R) = \frac{GMm_0}{R^2 m_0} = \frac{GM}{R^2}$

$I_{inside} (R>r) = 0$

Q Gravitational field intensity of solid sphere



$I_{outside}(r>R) = \frac{GMm_0}{r^2 m_0} = \frac{GM}{r^2}$

$I_{surface}(r=R) = \frac{GM}{R^2}$

$I_{inside}(R>r) = -\frac{GMm_0}{R^3 m_0} r \Rightarrow -\frac{GMr}{R^3}$

Q The G.F. on a body of mass 1.5 kg situated at a point is 45 N. The G.F.I. at that point is.

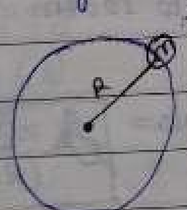
Ans $I = \frac{F}{m} = \frac{45}{1.5} = 30\,N$

Q Two point masses having mass m and 2m are placed at distance d. The point on the line joining point masses, where gravitational field intensity is zero will be at distance.

Ans $\frac{d}{\sqrt{n}+1} = \frac{d}{\sqrt{2}+1}$ from point mass 'm'.

Q Acceleration due to gravity at the surface of earth.

m_e = Mass of earth   R = Radius of earth



a = g (acc^n due to ~~ground~~ gravity)

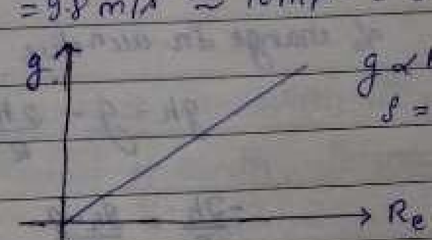
$F = \frac{GMm}{R^2}$

$\boxed{g} = a = \frac{F}{m} = \frac{GM_e}{\boxed{R^2}} = 9.8\,m/s^2 \approx 10\,m/s^2 = 32\,ft/s^2$

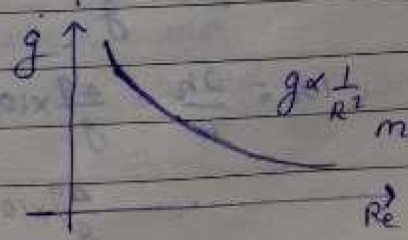
does not depend on mass of object

$g = \frac{G \rho \frac{4}{3}\pi R^{3}}{R^2}$

$\boxed{g} = \frac{4}{3} G \pi \rho \boxed{R}$



$g \propto R_e$
ρ = constant



$g \propto \frac{1}{R^2}$
mass = constant.

$g = \frac{GM}{R^2}$ $\boxed{gR^2 = GM}$ mR ratta

Weight $= mg$ = unit (N) $W_{moon} = \frac{mg_{earth}}{6}$

mass

ACCELERATION DUE TO GRAVITY ABOVE THE SURFACE OF EARTH

A point mass m at a height h above the surface of the earth

$F = \frac{GMm}{(R+h)^2}$

$g_h = \frac{GM}{(R+h)^2}$

$g_h = \frac{gR^2}{(R+h)^2}$

$g_{(h)} = \frac{gR^2}{(R+h)^2}$

Case-1: if h = 0 to 20 km

$h + R \cong R$

$\boxed{g_h = g_0}$

Case-2 if h 20km to 500 km

$g_h = g\left(\frac{R+h}{R}\right)^{-2}$

$g_h = g\left(1 + \frac{h}{R}\right)^{-2}$

By Binomial approximation

$(1+x)^n = 1 + nx$

when $x <<<< 1$

$\therefore \boxed{g_h = g\left(1 - \frac{2h}{R}\right)}$ for h = 20 to 500km

$\boxed{h = 800km} \longrightarrow$ formula $= \boxed{g_h = \frac{gR^2}{(R+h)^2}}$ (Always valid)

% change in accn due to gravity

$g_h = g - \frac{2hg}{R}$

$-\frac{2h}{R} = \frac{g_h - g}{g}$

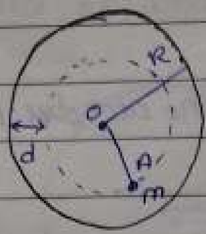
$100 \times -\frac{2h}{R} = \frac{\Delta g}{g} \times 100$

$\therefore \frac{\Delta g}{g} \times 100 = -\frac{2h}{R} \times 100$

LIMITED EDITION

ACCELERATION DUE TO GRAVITY BELOW THE SURFACE OF EARTH

A point mass m at A which is present at depth d below the surface of earth



$r = R-d$

$F = \frac{GMm(R-d)}{R^3}$

$g_d = \frac{GM(R-d)}{R^3}$

$g(d) = \frac{gR^2(R-d)}{R^3} = \frac{g(R-d)}{R} = \boxed{g\left(1-\frac{d}{R}\right)}$ → MR ratta.

→ Always applicable.

$g = \frac{g(R-d)}{R} \Rightarrow \frac{gr}{R}$

$g(d) = g - \frac{gd}{R}$

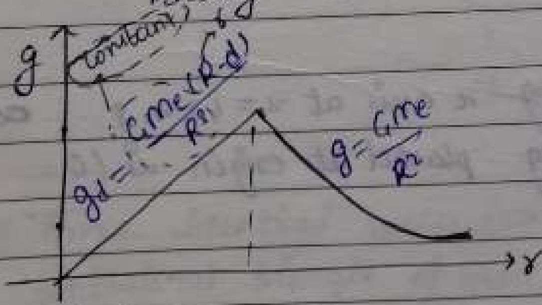
$\frac{g_d - g}{g} = \frac{-d}{R}$

$\Rightarrow \frac{\Delta g}{g} \times 100 = -\frac{d}{R} \times 100$

$g_d = g\left(1-\frac{d}{R}\right)$ → Always valid

$g_h = g\left(1-\frac{2h}{R}\right)$

$\Rightarrow \frac{gR^2}{g(R+h)^2}$ → Always valid



Q A body weighs 144 N at the surface of earth. When it is taken to a height of h = 3R, where R is radius of earth, it would weigh.

Ans $g(h) = \frac{gR^2}{(R+h)^2} = \frac{gR^2}{16R^2}$

$mg = 144$

$m = \frac{144}{g}$

weight at height h = $mg(h)$

$\Rightarrow \frac{\cancel{144}^{9}}{\cancel{g}} \times \frac{g}{\cancel{16}} = 9N$ **Ans**

LIMITED EDITION

Q The Gravitational constant depends upon
1) Size of the body
2) Gravitational mass
3) Distance b/w the bodies
~~4)~~ None of these

Q Two planets have same density but different radii. The accn due to gravity would be.
a) Same on both planets
b) Greater on the smaller planet.
~~c)~~ Greater on the larger planet
d) Dependent on the distance of planet from the sun.

$g = \frac{Gm}{R^2}$

$g = \frac{4}{3}\pi G \rho R$

$g \propto R$

Q If the radius of earth shrinks by 1.5% (mass remaining same), then the value of gravitational acceleration ~~would~~ changes by.

Ans For small change

$$g = \frac{G\,m}{R^2} \quad m \rightarrow \text{constant}$$

$$\frac{\Delta g}{g} \times 100 = -2\left[\frac{\Delta R}{R} \times 100\right]$$ We have to find % change

$$\Rightarrow -\cancel{2} \times \frac{-3}{\cancel{2}} \times 100 = 3\%$$

Q Mass particles of 1kg each are placed along x axis at $x = 1, 2, 4, 8 \ldots \infty$. Then gravitational force on a mass of 3kg placed at origin is (G = universal gravitational constant)

Ans

3kg

1 2 8 4 8

~~$F = 3Gm^2$~~

$$F = 3G + \frac{3G}{4} + \frac{3G}{16} \ldots$$

$$F = 3G\left(1 + \frac{1}{4} + \frac{1}{16} + \cdots\right)$$

$$\cancel{3}G \times \frac{4}{\cancel{3}} \qquad \hookrightarrow \text{sum} = \frac{a}{1-r} = \frac{1}{1-\frac{1}{4}} = \frac{4}{3}$$

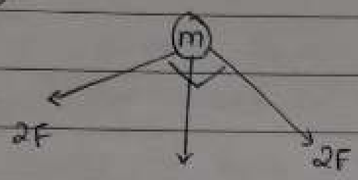
$F = 4G$ Ans

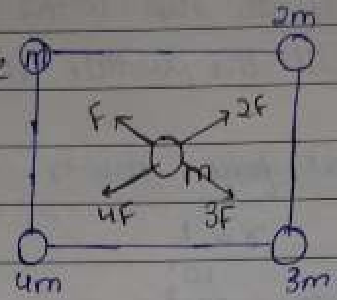
Q → Four particles of masses m, 2m, 3m and 4m are kept in sequence at corners of a square of side a. The mgn of G.F. acting on a particle of mass m placed at the centre of the square will be:

Ans

Net force on the particle

⇒





$F_{net} = \sqrt{2}\,(2F)$

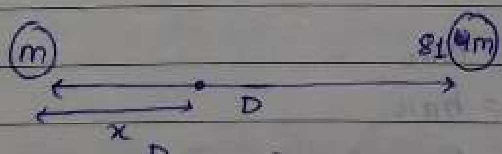
$\sqrt{2} \times 2 \times \frac{Gm^2}{\frac{a^2}{2}} \Rightarrow \frac{4\sqrt{2}\,Gm^2}{a^2}$ Ans

Q The tidal waves in the seas are primarily due to

→ Gravitational effect of the moon on the earth

Q If the distance between the centres of earth and moon is D and mass of earth is 81 times that of moon. At what distance from centres of earth gravitational field will be zero.

Ans



$x = \frac{d}{\sqrt{n}+1} = \frac{D}{9+1} = \frac{D}{10}$

Distance from earth (81m) $= \frac{9D}{10}$

Q The value of 'g' reduces to half of its value at surface of earth at a height 'h' then -

Ans $\cancel{g_h = g\left(1 - \frac{2h}{R}\right)}$ $g_h = \frac{gR^2}{(R+h)^2}$

$\cancel{\frac{g}{2} = \frac{g}{2} = \frac{gR^2}{(R+h)^2}}$ $\frac{g}{2} = \frac{gR^2}{(R+h)^2}$

LIMITED EDITION

~~$R^2+h^2+2Rh = 2R^2$~~

~~$h^2+2Rh = R^2$~~

~~$h = R^2-2Rh$~~

$$\frac{1}{\sqrt{2}} = \frac{R}{R+h}$$

$R+h = \cancel{\sqrt{R}}\sqrt{2}R$

$R-\sqrt{2}R = -h$

$h = \sqrt{2}R-R$

$h = R(\sqrt{2}-1)$ Ans

Q At some planet 'g' is $1.96\ m/s^2$. If it is safe to jump from a height of 2m on earth, then what should be corresponding safe height for jumping on that planet

Ans $v = \sqrt{2ah} = \sqrt{2\times 9.8\times 2}$

At another planet

$\sqrt{\cancel{2}\times 9.8\times 2} = \sqrt{\cancel{2}\times 1.96\times x}$

$\Rightarrow \overset{5}{\cancel{9.8}}\times 2 = \cancel{1.96}\times x$

$x = 10\ m$

Q If the earth stops rotating suddenly, the value of 'g' at a place other than poles would

Ans Increase

Q When the radius of earth is reduced by 1% without changing the mass, then the acc^n of due to gravity will.

Ans $g = \frac{Gm}{r^2}$

$g \propto \frac{1}{r^2}$

$$\frac{dg}{g}\times 100 = -2\left(\frac{dr}{r}\times 100\right)$$

$\frac{dr}{r}\times 100 = -1\%$

$\therefore \frac{dg}{g}\times 100 = 2\%$ Ans (Increases by 2%)

LIMITED EDITION

Q Weight of a body of mass m decreases by 1% when it is raised to a height h above the earth's surface. If the body is taken to a depth h in a mine, then in its weight will

Ans $\frac{\Delta g}{g} = \frac{2h}{R} = 1\%$ $\quad \frac{h}{R} = \frac{1}{2}\%$

$\frac{\Delta g}{g} = \frac{d}{R} = ??$

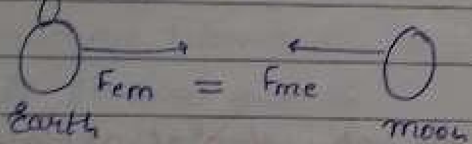
$\Rightarrow \because d = h \quad \therefore \frac{h}{R} = 0.5\%$ decrease

Q Acceleration due to gravity at earth's surface is g m/s². Find the effective value of acc due to gravity at a heigh of 32 km from sea level (R = 6400 km)

Ans $\rightarrow g_h = g\left(1 - \frac{2h}{R}\right) \Rightarrow g\left(1 - \frac{2\times 32}{6400}\right) = g\left(1 - \frac{1}{100}\right) = 0.99\,g$ m/s² Ans

Q The mass of the moon is 1% of mass of earth. The ratio of G.F. pull of earth on moon to that of moon on earth will be.

Ans



$\therefore$ Ratio = 1:1

Q Imagine a new planet having the same density as that of earth but its radius is 3 times bigger than the earth in size. If the accn due to gravity on the surface of earth is g and that on the surface of the new planet is (g') then.

Ans For same density $g = \frac{4}{3}\pi \rho G r$

$\therefore g \propto r$

$\therefore$ if $r\uparrow$ by 3 times then $g' = 3g$ Ans

Q If density of a planet is double that of earth and the radii 1.5 times that of earth, the accn due to gravity on the surface of planet is.

Ans $g = \frac{4}{3}\pi \rho G R$

$\Rightarrow g \propto \rho R \Rightarrow 1.5\times 2 = 3$ times that on the surface of earth

LIMITED EDITION

Q At what height have the surface of earth of "g" decreases by 2%? (Radius of the earth is 6400 km)

Ans $g_h = g(1 - \frac{2h}{R})$

$\frac{\Delta g}{g} \times 100 = \frac{2h}{R} \times 100$

$\cancel{2} = \frac{\cancel{2} \times h}{64\cancel{00}} \times 1\cancel{00}$ $h = 64\,km$

Q The value of g at the surface of earth is 9.8 m/s^2. Then the value of 'g' at a place 480 km above the surface of the earth will be nearly (radius of the earth is 6400 km)

Ans $g_h = (1 - \frac{2h}{R}) g_0$

$\Rightarrow g_0 \left(1 - \frac{2 \times 480}{6400}\right) g$

$\Rightarrow \left(1 - \frac{3}{20}\right) g$

$\Rightarrow \left(\frac{17}{\cancel{20}_{10}}\right) \times \cancel{9.8}^{4.9} = 8.33\,m/s^2$

Q If the change in the value of 'g' at a height 'h' above the surface of the earth is same as at a depth x below it, then (x and h being much smaller than the radius of earth)

Ans $\frac{\cancel{+}2h}{\cancel{R}} = \frac{\cancel{+}d}{\cancel{R}}$

$2h = d$

$2h = x$ Ans

Q The acceleration due to gravity on a planet is 1.96 m/s^2. If it is safe to jump to a height of 3 m on the earth, the corresponding height on the planet will be

Ans $H = \frac{v^2}{2g}$ constant ∴ $Hg_1 = H_2 g_2$

$3 \times \cancel{9.8}^{5} = H_2 \times \cancel{1.96}$

$H_2 = 15\,m$

OR

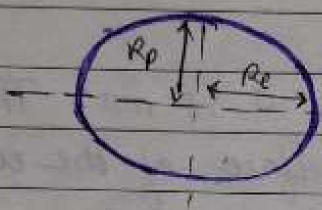
$\sqrt{2as} = v$

$\cancel{2} \times \cancel{9.8}^{5} \times 3 = \cancel{2} \times \cancel{1.96} \times x$

$x = 3 \times 5 = 15\,m$

Variation of 'g' due to shape of the Earth

In reality, the earth does not have pure spherical shape. It has oval shape. Its equatorial radius (R_e) is higher than its polar radius (R_p). (Approximately, $R_{eq} = R_p + 21$ km)

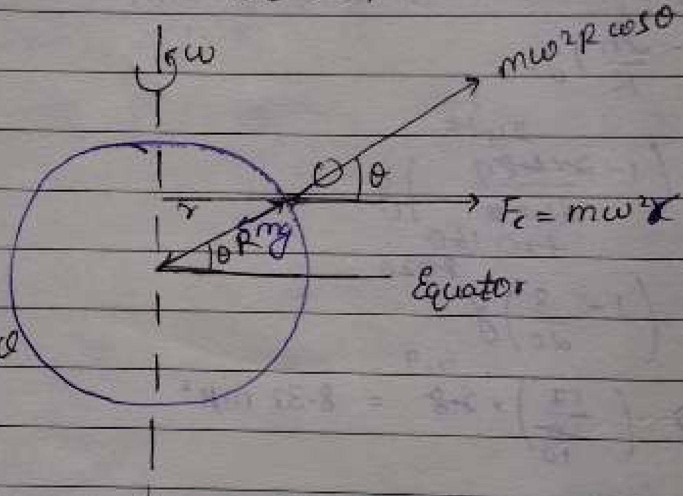


$R_e > R_p$

$g_e < g_P$

$m_e = m_P$ (Person's mass)

$W_E < W_P$



$F_{net} = mg - m\omega^2 R\cos\theta$

$F_{net} = mg - m\omega^2 r\cos\theta$

$\cancel{m}g_{eff} = \cancel{m}g - \cancel{m}\omega^2 r\cos\theta$

$g_{eff} = g - \omega^2 r\cos\theta$

$\cos\theta = \frac{r}{R}$

$r = R\cos\theta$

$\therefore g_{eff} = g - \omega^2 R\cos^2\theta$

$\therefore$ On poles there is no effect in acc^n due to gravity due to earth's rotation but has effect on equator.

At equator $\theta = 0°$

$g_{eff} = g - \omega^2 R$

At Pole $\theta = 90°$

$g_{eff} = g$

$\omega = \left(\frac{2\pi}{T}\right)$ $T = (24$ hours$)$

$g_{eff} = g - \omega^2 R$

$g_{eff} - g = -\omega^2 R$

$100 \times \frac{\Delta g}{g} = -\frac{\omega^2 R}{g} \times 100$

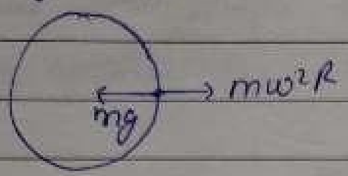
$\frac{\Delta g}{g} \times 100 = 0.34\%$ (% decrease in acc^n due to gravity due to rotation of earth)

LIMITED EDITION

★

Q The radius of earth is 6400 km and $g = 9.8\ m/s^2$. If the body placed at the equator has to become weightless the earth should make one complete rotation in

Ans $\cancel{m}g = \cancel{m}\omega^2 R$



$g = \omega^2 R$

$\sqrt{\frac{g}{R}} = \omega = \frac{2\pi}{T}$

$T = 2\pi\sqrt{\frac{R}{g}} = 2\pi\sqrt{\frac{640\cancel{0} \times 1000\, m}{1\cancel{0}\, m/s^2}} = 2\pi \times 800$

$\Rightarrow 1600 \times \frac{22}{7}$ s

$\Rightarrow \frac{160\cancel{0} \times 22}{3\cancel{60} \quad 7}$ min $= \frac{\overset{84}{\cancel{1760}}}{\cancel{21}}$ min $= 84$ min $= \frac{84}{60} = 1.4$ hour.

When 'ω' is increased by $\boxed{17}$ times of its initial value this happens

Q At a height above the surface of earth equal to the radius of the earth, the value of g will be.

Ans $g_h = \frac{gR^2}{(R+h)^2} = \frac{gR^2}{4R^2} = \frac{g}{4}$ Ans

Q The effect of rotation on the value of accⁿ due to gravity is.

a) g is maxᵐ at the equator and minᵐ at poles.

b) g is minᵐ at the ~~po~~ equator and maxᵐ at poles.

c) g is maxᵐ at both places

d) g is minᵐ at both places.

Q Which of the following statements are true about accⁿ due to gravity,

(i) 'g' decreases in moving away from the centre if $r > R$.

(ii) 'g' decreases in moving away from the centre if $r < R$.

(iii) 'g' is zero at the centre of earth

(iv) 'g' decreases if earth stops rotation on its axis.

Q At what depth below the surface of the earth, the value of g is the same as that at a height of 5 km?

Q If earth suddendly stop rotating, then the wt. of an object of mass m at equator will (ω is angular speed of earth and R is its radius)

Ans At pole weight remains same

At equator $g_{eff} = g - \omega^2 R$

$g_{eff} = g$

$mg_{eff} = mg$

$\therefore$ wt. increases by $m\omega^2 R$

Q During motion of a man from equator to pole of earth, it weight will change by what %

Ans weight will increase by 0.34%

Q As we go from the equator to the poles, value of 'g'

Ans increases

Q How much deep inside the earth (Radius R) should a man go, so that his wt. becomes $\frac{1}{4}$th of that on earth's surface?

As $g_d = g\left(1 - \frac{d}{R}\right)$

$\frac{1}{4} g = g\left(\frac{R-d}{R}\right)$

$R = 4R - 4d$

$d = \frac{3R}{4}$ Ans

Q The height at which the accn due to gravity becomes $\frac{g}{9}$ (where g = accn due to gravity on the surface of the earth) in terms of R, and radius of the earth is.

Ans $\frac{g'}{g} = \frac{g R^2}{(h+R)^2}$

$\frac{1}{3} = \frac{R}{R+h}$

$R+h = 3R$

$h = 2R$ Ans

LIMITED EDITION

Ans $2h = x$

$x = 10\,km$

Q The moon's radius is $\frac{1}{4}$ that of a earth and its mass is $\frac{1}{80}$ times that of the earth. If g represents the accn due to gravity on the surface of the earth, then that on the surface of moon is.

Ans $g = \frac{Gm}{R^2} = \frac{1}{\cancel{5}\,80 \times \frac{1}{16}} = \frac{1}{5}$ times $\Rightarrow \frac{g}{5}$

Q Radius of earth is around 6000 km. The weight of the body at height of 6000 km from earth surface becomes.

Ans = if $R = h$

then $g_{(h)} = \frac{gR^2}{4R^2} = \frac{g}{4} \therefore w' = \frac{w}{4}$

Q If the density of the earth is doubled keeping its radius constant then accn due to gravity will be ($g = 9.8\ m/s^2$)

Ans $g = \frac{4}{3}\pi \rho G r$

$\therefore$ if ρ is doubled g is also doubled $\therefore$ Ans $\rightarrow 19.6\,m/s^2$

Q Imagine a new planet having the same density as that of earth but it is 3 times bigger than the earth in size. If the accn due to gravity on the surface of earth is g and that on the surface of the new planet is g': then.

Ans $g = \frac{4}{3}\pi \rho G R \rightarrow$ 3 times bigger

$\therefore g' = 3g$

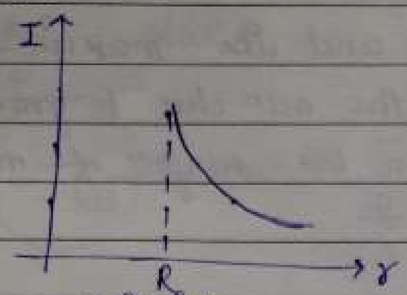
Q Altitude at which accn due to gravity decreases by 0.1 % approximately (Radius of earth: 6400 km)

Ans $\frac{\Delta g}{g} \times 100 = \frac{2h}{R} \times 100$

$\Rightarrow \frac{0.1}{100 \times 2} \times R = h \Rightarrow h = 0.001 \times \frac{\overset{3200}{\cancel{6400}}}{\cancel{2}} = 3.2\,km.$

Q Which one of the following plots represent the variation of gravitational field of a particle with distance r due to thin spherical shell of radius R is:

Ans



Ans

Q If the spinning ~~speeding~~ speed of the earth is increased, then the weight of the body at equator

Ans decreases

Q What will happen to the weight of the body at the south pole if the earth stops rotating about its polar axis.

Ans No change.

NOTES

1. If we have to find the time period of rotation of the earth so that a body on equator feels weightless then make $g - \omega^2 R = 0$ or $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{R}{g}} = 84.6$ min. This also mean that the earth will have to rotate 17 times of its present rotation speed $\left(\frac{24 \text{ hours}}{84.6 \text{ min}} = 17\right)$ for a body at equator to become weightless.

2. If the angular speed of the earth increases, the value of effective g will decrease at all the places except at poles.

3. Since earth rotates from west to east, we project our rockets also in west to east direction so that effective $g' = g - \omega^2 R$ on the rocket is less

★ Q If Accn due to gravity decreases by 36% at a height h from surface, then find h ??

Ans $g(h) = \frac{gR^2}{(R+h)^2}$

$$\Rightarrow \frac{64}{100} \times \cancel{g} = \frac{\cancel{g}R^2}{(R+h)^2}$$

$$\frac{8}{10} = \frac{R}{R+h}$$

$$8R + 8h = 10R$$

$$\cancel{8}4h = \cancel{2}R$$

$$h = \frac{R}{4} \quad \text{Ans}$$

GRAVITATIONAL POTENTIAL ENERGY

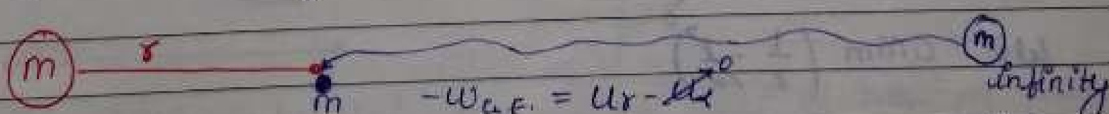
\# Gravitational potential energy is not defined

\# Gravitational Potential does not have unique value it depends upon reference point

$$-W_{C.F.} = \Delta U$$

$$-W_{A \text{ to } B} = U_B - U_A$$

$-$ve work done by Gravitational force in bringing a mass m from infinity to the point in gravitational field is called Gravitational potential energy at that.



$$-W_{G.F.} = U_r - \cancel{U_\infty}^{\,0}$$

$U_\infty = 0$

Work done by external force in bringing a mass m from infinity to the point without change in K.E. is called Gravitational PE at that point.

Work energy theorem

$W_{all\ forces} = \Delta K.E.$

$W_{C.F} + W_{N.C.F.} = \Delta K.E.$

if $W_{N.C.F} = 0$

$W_{C.F.} = \Delta K.E.$

$-\Delta U = \Delta K.E$

$\Delta U + \Delta K.E = 0$

$\Delta(K.E. + U) = 0$

$K.E + U = \text{constant}$

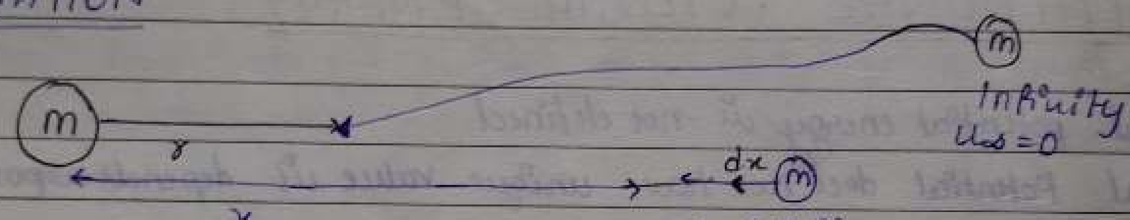
$W_{C.F.} + W_{N.C.F.} = \Delta K.E.$

$(K.E. = \text{constant})$

$-\Delta U + W_{N.C.F.} = 0$

$W_{N.C.F} = \Delta U$

DERIVATION



$F = \frac{GMm}{x^2}$

$\int dW = \int F.dx$

$W = \int_{\infty}^{r} \frac{GMm}{x^2}\, dx$

$W = -[GMm] \int_{\infty}^{x} \frac{dx}{x^2}$

due to decrease in x

$W = +[GMm] \left(\frac{1}{x}\right)_{\infty}^{r}$

$W = GMm \left(\frac{1}{r} - \cancel{\frac{1}{\infty}}^{0}\right)$

$W = \frac{GMm}{r}$

$W = -\Delta U$

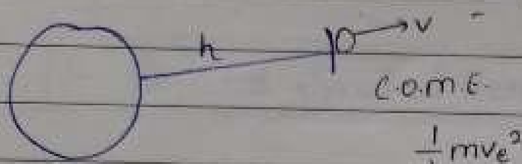
$\frac{GMm}{r} = -[U_f - \cancel{U_\infty}^{0}]$

$U_f = -\frac{GMm}{r}$

Escape Velocity

→ Depends upon mass of planet and radius.

→ does not depend upon mass of object and angle of projection



infinity
$v=0$
$u_\infty=0$

C.O.M.E.

$$\frac{1}{2}mv_e^2 - \frac{GMm}{R+h} = 0+0$$

$$\frac{1}{2}\cancel{m}v_e^2 = \frac{GM\cancel{m}}{R+h}$$

$$v_e = \sqrt{\frac{2GM}{R+h}}$$

$h=0$

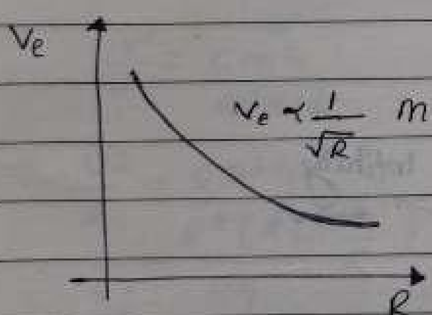
$$v_e = \sqrt{\frac{2GM}{R}}$$

$$V_e = \sqrt{\frac{2GM}{R}} = \sqrt{2gR} = \sqrt{\frac{2G\rho\frac{4}{3}\pi R^{3}}{R}}$$

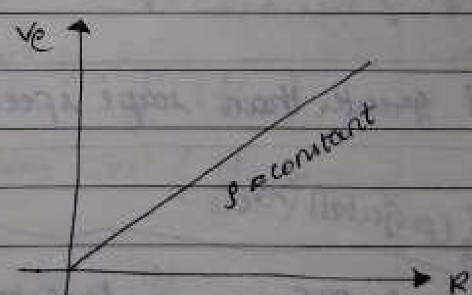
$$\Rightarrow \sqrt{2G\rho\frac{4}{3}\pi R^2} = \sqrt{\frac{8}{3}\pi G\rho R^2} = \sqrt{\frac{8}{3}\pi G\rho R^2}$$

$$\boxed{v_e = \sqrt{\frac{2GM}{R}} = \sqrt{2gR} = \sqrt{\frac{8}{3}\pi G\rho R^2}}$$

$$v_e \propto R\sqrt{\rho}$$



$v_e \propto \frac{1}{\sqrt{R}}$ $\quad m =$ constant



Q The total mechanical energy of an object of mass m projected from surface of earth with ~~esc~~ speed is

→ zero

Q The escape velocity of a body from earth 11.2 km/s. Assuming the mass and radius of the earth to be about 81 times and 4 times the mass and radius of moon, the escape velocity in km/s from the surface of the moon will be.

Ans $V_{e(m)} = \sqrt{\frac{G m_m \times 2}{r_m}} = \sqrt{\frac{2G \frac{1}{81} m_e \, 4}{r_e}} = \frac{2}{9}\sqrt{\frac{2Gm_e}{r_e}} = \frac{2}{9} \times 11.2 = \frac{22.4}{9} = 2.48 \text{ km/s}$

Q The atmosphere on a planet is possible only if (where V_{rms} is root mean square speed of gas molecules of gas molecules on planet and V_e is escape speed on its surface)

(a) $V_{rms} = V_e$ (b) $V_{rms} > V_e$

(c) $V_{rms} \leq V_e$ (d) $V_{rms} < V_e$

$V_{rms} = 6$ km/s
$V_e = 11.2$ km/s (for earth)

Q If M is mass of a planet and R is its Radius then in order to become black hole (c is speed of light)

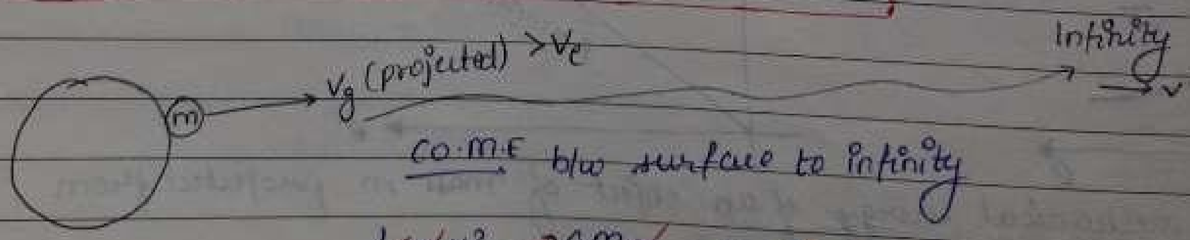
(a) $\sqrt{\frac{GM}{R}} \leq c$ (b) $\sqrt{\frac{GM}{2R}} \geq c$

(c) $\sqrt{\frac{2GM}{R}} \geq c$ (d) $\sqrt{\frac{2GM}{R}} \leq c$

$V_e \geq c$

Black hole $\rightarrow$ strongest gravitational field

If Projected speed greater than scape speed



CO.M.E b/w surface to infinity

$\frac{1}{2} m V_g^2 - \frac{GMm}{R} = 0 + \frac{1}{2} m V_{space}^2$

$V_g^2 - \frac{2GM}{R} = V_{space}^2$

$\because \frac{2GM}{R} = V_e^2$

$V_g^2 - V_e^2 = V_{space}^2$

$V_{space} = \sqrt{V_g^2 - V_e^2}$ If $V_g = V_e$, $V_{space} = 0$

Q. A body is thrown with a velocity equal to n times the escape velocity (v_e). Velocity of the body at a large distance away will be.

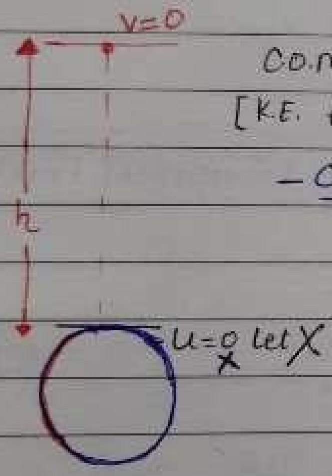
Ans → $V_{(s)} = \sqrt{v_g^2 - v_e^2}$

$\Rightarrow V_{(s)} = \sqrt{n^2 v_e^2 - v_e^2}$

$\Rightarrow V_{(s)} = \sqrt{(n^2 - 1) v_e^2}$

$V_{(s)} = V_e \sqrt{(n^2 - 1)}$ Ans

If Projected Speed is less than Escape speed



C.O.M.E.

$[K.E. + U]_{\text{earth surface}} = (K.E + U)_{\text{at height}}$

$$-\frac{GM\cancel{m}}{R} + \frac{1}{2}\cancel{m}v^2 = 0 - \frac{GM\cancel{m}}{R+h}$$

$$\frac{v^2}{2} = \frac{GM}{R} - \frac{GM}{R+h}$$

$$\frac{v^2}{2} = GM\left(\frac{1}{R} - \frac{1}{R+h}\right)$$

$$\frac{v^2}{2} = GM\left(\frac{\cancel{R}+h-\cancel{R}}{R(R+h)}\right)$$

$$\frac{v^2}{2} = \frac{GMh}{R(R+h)}$$

$$\frac{v^2}{2} = \frac{gR^2 h}{R^2\left(1 + \frac{h}{R}\right)}$$

$$\frac{v^2}{2} = \frac{gh}{1 + \frac{h}{R}}$$

$$v = \sqrt{\frac{2gh}{1 + \frac{h}{R}}}$$

if $h <<<< R$

$$v = \sqrt{2gh}$$

$$\frac{v^2}{2} = \frac{2GM}{R}\left(\frac{H}{R+H}\right)$$

$$v^2 = V_e^2\left(\frac{H}{R+H}\right)$$

$$\frac{v^2}{V_e^2} = \frac{H}{R+H}$$

$$\frac{V_e^2}{v^2} = \frac{R+H}{H}$$

$$\frac{V_e^2}{v^2} = \frac{R}{H} + 1$$

$$\frac{R}{H} = \frac{V_e^2}{v^2} - 1$$

$$\boxed{H = \frac{R}{\left(\frac{V_e^2}{V_g^2} - 1\right)}} \Bigg/ \frac{Rv^2}{2g - \frac{v^2}{R}}$$

1) Dimension

2) If $V_e = V_g$

$h = \infty$

$\frac{V_g^2}{V_e^2}$ × Height cannot be negative

as $V_g < V_e$

Q A body is projected ~~with~~ vertically upwards with a speed of $\sqrt{\frac{GM}{R}}$ (M is mass and R is radius of earth). The body will attain a height of

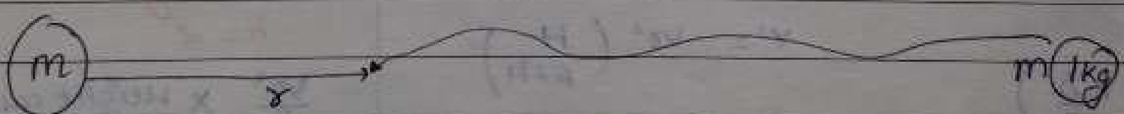
Ans
$$H = \frac{R}{\frac{V_e^2}{V_g^2} - 1} = \frac{R}{\frac{\frac{2GM}{R}}{\frac{GM}{R}} - 1} \Rightarrow \frac{R}{2-1} = R \text{ Ans}$$

Q A body is projected vertically upwards with a speed of $\sqrt{\frac{GM}{2R}}$ (M is mass and R is radius of earth) The body will attain a height of

Ans
$$H = \frac{R}{\frac{V_e^2}{V_g^2} - 1} = \frac{R}{\frac{\frac{2GM}{R}}{\frac{GM}{2R}} - 1} = \frac{R}{3}$$

GRAVITATIONAL POTENTIAL

-ve work done by Gravitational force in bringing unit mass from infinity to that point is called Gravitational Potential



$$U = -\frac{GMm}{r}$$

Gravitational Potential Energy per unit mass is called gravitational potential.

$$V = \frac{U}{m} = \frac{-GM}{r}$$

$$\vec{F}_{gF} = -\frac{dU}{dr}$$

$$\frac{\vec{F}}{m} = \frac{-dU/m}{dr}$$

$$\vec{I} = -\frac{\partial V}{\partial r}\hat{r}$$

$\frac{\partial V}{\partial r}$ → Potential ~~energy~~ gradient

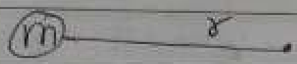
$$\vec{I} = -\left(\left(\frac{\partial V}{\partial x}\right)\hat{i} + \left(\frac{\partial V}{\partial y}\right)\hat{j} + \left(\frac{\partial V}{\partial z}\right)\hat{k}\right)$$

$\vec{I} = \frac{-\partial v}{dr}$

$-\int I \, dr = \int dv$

$\Delta v = -\int \vec{I} \cdot \vec{dr}$

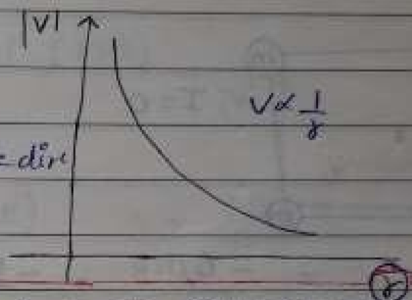
GRAVITATIONAL POTENTIAL OF POINT MASS



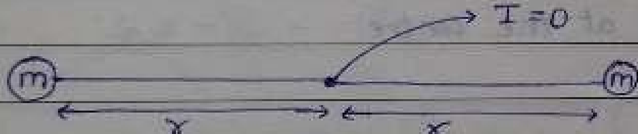
$V = -\frac{Gm}{R}$

↳ scalar

↳ does not have dir



GRAVITATIONAL POTENTIAL DUE TO COLLECTION OF POINT MASS



$V_p = -\frac{Gm}{r} - \frac{Gm}{r} = -\frac{2Gm}{r}$

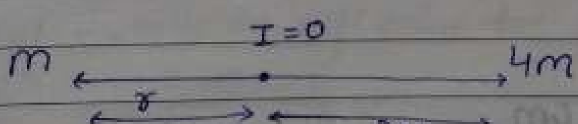
$I = -\frac{dv}{dr}$

If $v = 0$ then I may be zero

v = const^n then I must be zero.

Q Two point masses having mass m and 4m are placed at distance r. The gravitational potential at a point, where gravitational field intensity is zero is

Ans



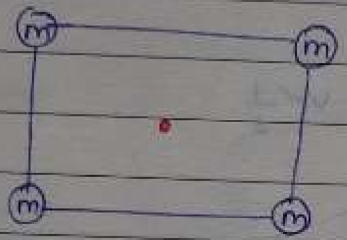
$x = \frac{r}{\sqrt{n}+1} = \frac{r}{3}$

$$V = \frac{-Gm}{r/3} - \frac{\overset{2}{\cancel{4}}Gm}{\frac{\cancel{2}r}{3}}$$

$$V = -\frac{3Gm}{r} - \frac{6Gm}{r}$$

$$V = -\frac{9Gm}{r} \quad \underline{Ans}$$

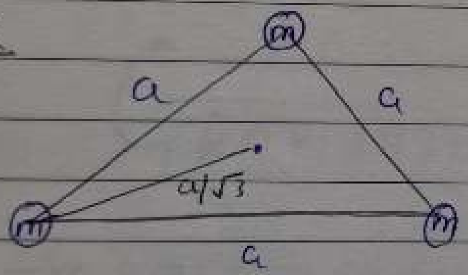
Q Find potential at the centre of square



$I = 0$

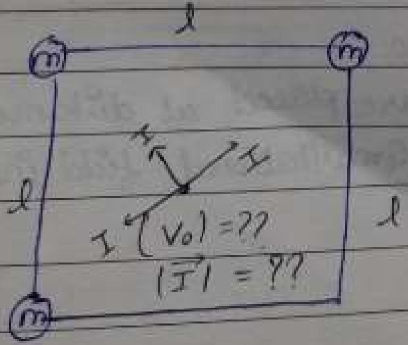
$$V_0 = \frac{-Gm4}{\frac{l}{\sqrt{2}}} = \frac{-4\sqrt{2}Gm}{l}$$

Q Find potential at the centre



Ans $V = -\frac{Gm}{\frac{a}{\sqrt{3}}} \times 3 = -\frac{3\sqrt{3}Gm}{a}$

Q



$$V_0 = -\frac{Gm \times 3}{\frac{l}{\sqrt{2}}} = -\frac{3\sqrt{2}Gm}{l}$$

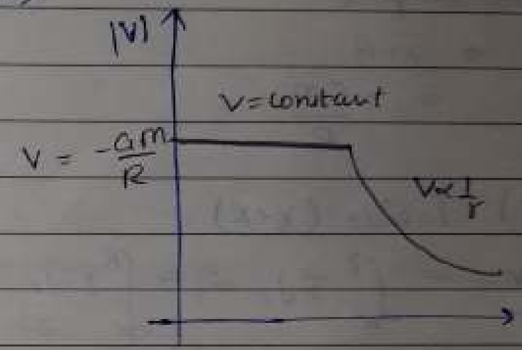
$$\vec{I}_{net} = \vec{I} \text{ toward } m \quad = \frac{\cancel{3Gm}}{\cancel{l^2}} \quad \frac{2Gm}{l^2} \quad \frac{Gm}{\left(\frac{l}{\sqrt{2}}\right)^2}$$

GRAVITATION POTENTIAL

Potential due to a uniform thin spherical shell.
(Mass M and radius R)

Case-1 Potential at outside Point $(r>R)$

$$V_P = -\frac{GM\cancel{m_0}}{r\,\cancel{m_0}} = -\frac{GM}{r}$$



Case-2 Potential at surface $(r=R)$

$$V_{surface} = \frac{-GM}{R}$$

Case-3 Inside $(R>r)$

$$V_{P_r} - V_\infty = -\int_\infty^P I\cdot dr$$

$$V_{P_r} - 0 \Rightarrow -\int_\infty^R I'dr - \int_R^r \cancel{I_{inside}}^{\,0}\, dr$$

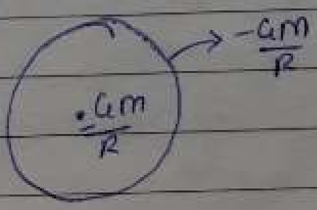
$$V_{inside} = \frac{-GM}{R} = \text{at surface}$$

$$V_{\text{centre of hollow sphere}} = \frac{-GM}{R} \qquad I_{inside} = 0$$

MR* problem

If Gravitational potential at the centre of hollow sphere sp^r is assumed to be zero then potential at surface of Hollow sp^r will be.

Ans Potential depends on ref^r point but potential diff^r does not depend on ref^r.



$V_s - V_0 = 0$

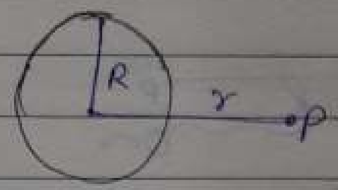
POTENTIAL DUE TO UNIFORM SOLID SPHERE

Let us consider a uniform solid of mass M and radius R

Case - 1

When P is outside the sphere $(r \geq R)$

$$V_P = -\frac{GM}{r}$$



Case - 2 At surface

$\Rightarrow r = R$

$\Rightarrow -\frac{GM}{R}$

(Case - 3) Inside $(r < R)$

$$\Delta V = -\int_{\infty}^{r} I\,dr = -\int_{\infty}^{R} I'\,dr - \int_{R}^{r} I\,dr$$

$$V_r - \cancel{V_\infty}^{0} = -\frac{GM}{R} - \int_R^r \frac{GM}{R^3} \times r\,dr$$

$$V_r = -\frac{GM}{R} - \frac{GM}{R^3}\int_R^r r\,dr$$

$$V_r = -\frac{GM}{R} + \frac{GM}{R^3}\int_R^r r\,dr$$

$$V_r = -\frac{GM}{R} + \frac{GM}{R^3}\left(\frac{r^2}{2}\right)_R^r$$

$$V_r = -\frac{GM}{R} + \frac{GM}{2R^3}\left[r^2 - R^2\right]$$

$$V_r = -\frac{GM}{R} + \frac{GM}{2R^3}\left[r^2 - R^2\right)$$

$$V_r = \frac{GM}{2R^3}\left[r^2 - R^2 - 2R^2\right]$$

$$\boxed{V_r = \frac{GM}{2R^3}\left(r^2 - 3R^2\right)}$$

If $r = 0$

$V = -\frac{3GM}{2R}$

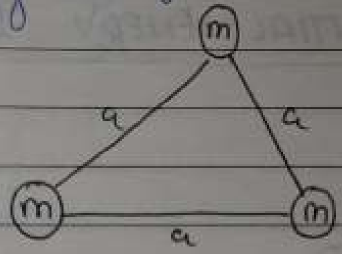
$V_0 = -3GM$

If $r = R$

at surface

$V_s = -\frac{GM}{R}$

Q Find potential energy of the system

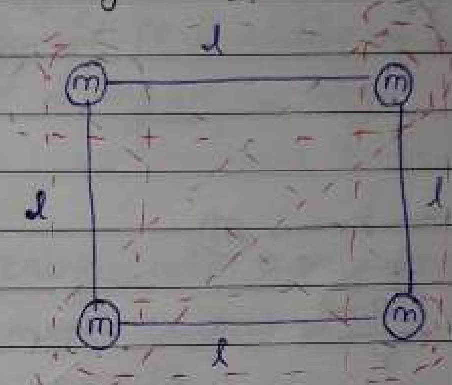


Ans $U = -\frac{Gm^2}{a} - \frac{Gm^2}{a} - \frac{Gm^2}{a}$

$\Rightarrow -\frac{3Gm^2}{a}$

Q Find potential energy of the system ??

Ans $-\frac{Gm^2}{l}4 - \frac{2Gm^2}{\sqrt{2}l}$



MR*

Total no. of terms for n mass $= \frac{n(n-1)}{2}$

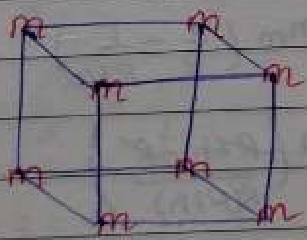
To count no. of pairs

Q Find potential energy of the system

Ans No. of terms $= 6$

$U = -\frac{Gm^2}{a}3 - \frac{Gm^2}{2a}\times 2 - \frac{Gm^2}{3a}$

Q MR*



Find Potential energy of the system

Ans Total terms $= \frac{8(7)}{2} = 28$

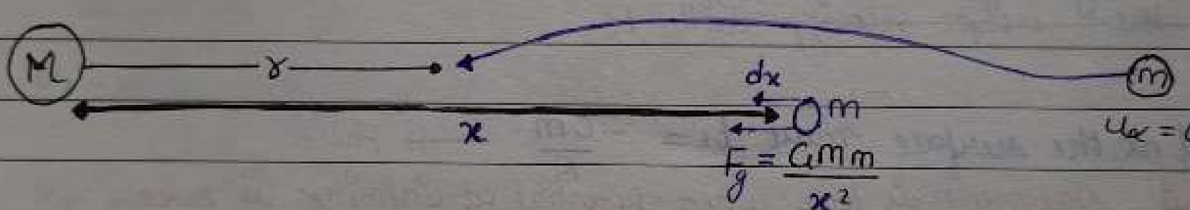
$U = -\frac{12Gm^2}{l} - \frac{12Gm^2}{\sqrt{2}l} - \frac{4Gm^2}{\sqrt{3}l}$

LIMITED EDITION

GRAVITATIONAL POTENTIAL ENERGY

Gravitational potential energy of two point masses

Grav. Potential energy is not defined for a single mass for there is no gravitational field generated इसलिए हमने एक point mass लिए और उसे इतनी दूर रखा w.r.t other particle at infinity such that Gravitational Potential energy of the system is zero. अब अगर मुझे दो mass वाले system का combined gravitional Potential energy चाहिए तो मैं एक हका से force लगाऊंगा infinity मे रखे हुए mass पर और कोई r distance की दूरी पर उनका Gravitational potential energy calculate करूंगा



$$\Delta U = -W_{C.F.}\ (\text{gravitation})$$

$$\Delta U = -\int_{x=\infty}^{r} \frac{GMm}{x^2}\, dx$$

$$\Delta U = GMm \int \frac{1}{x^2}$$

$$U_r - \cancel{U_\infty}^{\,0} = GMm \left[-\frac{1}{x}\right]_\infty^r$$

$$U_r = -GMm\left[\frac{1}{r} - \cancel{\frac{1}{\infty}}^{\,0}\right]$$

$$U_r = -\frac{GMm}{r} \quad \text{Ans}$$

$$U = \frac{-Gm_1m_2}{r}$$

Scalar, no direction

Potential energy is less than zero

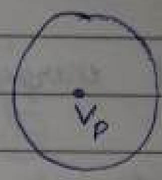
when Work done by C.F. is +ve the U↓

If Potential energy at infinity is zero then -ve potential energy means stable system

$U = +ve \longrightarrow$ Free system

Q If Potential at the surface of earth is assigned zero value, then potential at centre of earth will be (Mass = M, Radius = R)

Ans $V_P = \cancel{V_\infty}^{\,0} - \int_{\infty}^{R} \cancel{I}^{\,0}\, dr - \int_{R}^{0} I\, dr$



$V_P = + \int_{R}^{0} \frac{GM}{R^3} r\, dr$

due to dr

$V_P = \frac{GM}{R^3} \left[\frac{r^2}{2}\right]_R^0$

$V_P = \frac{GM}{R^3}\left(0 - \frac{R^2}{2}\right)$

$\Rightarrow -\frac{GM}{R^3}\frac{R^2}{2} \Rightarrow \frac{-GM}{2R}$

Q ~~Potential energy of a 3 kg body at the surface of a planet is -54 J, then escape velocity will be.~~

~~Ans~~

Potential at the surface must be $= \frac{-GM}{R} \longrightarrow$ False

this statement is true when potential at infinity is zero.

But the potential difference between any two points is constant

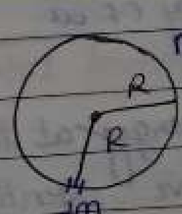
$\therefore V_{surface} - V_{centre} = -\frac{GM}{R} + \frac{3GM}{2R} = \frac{GM}{2R}$

$\therefore$ For the above question

$\cancel{V_s}^{\,0} - V_{centre} = \frac{GM}{2R}$

$V_{centre} = -\frac{GM}{2R}$

POTENTIAL DUE TO RING

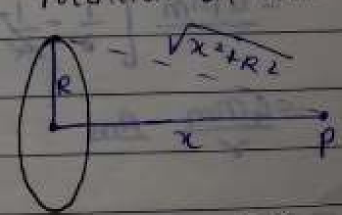


$\int dV = -\int \frac{G\, dm}{R}$

$V = -\frac{G}{R}\int dm$

$V = -\frac{GM}{R}$

Potential on the axis of ring



$V = \frac{-GM}{\sqrt{x^2+R^2}}$

If $x = 0$

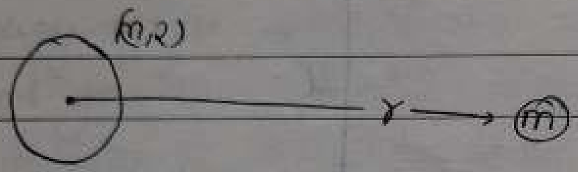
$V = -\frac{GM}{R}$

GRAVITATIONAL POTENTIAL ENERGY DUE TO SOLID SPHERE

Solid sphere

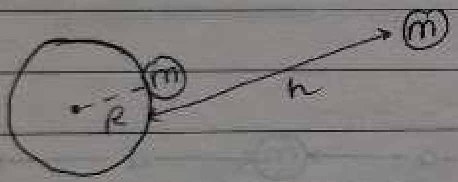
$$u = -\frac{GMm}{r}$$

Hollow sphere



$$u = -\frac{GMm}{r}$$

GRAVITATIONAL POTENTIAL ENERGY OF EARTH MASS SYSTEM



$$u_f = -\frac{GMm}{R+h}$$

$$U_{earth-mass} = -\frac{GMm}{R}$$

Change in P.E.

$$\Delta u = u_f - u_i$$

$$\Delta u \Rightarrow -\frac{GMm}{R+h} + \frac{GMm}{R}$$

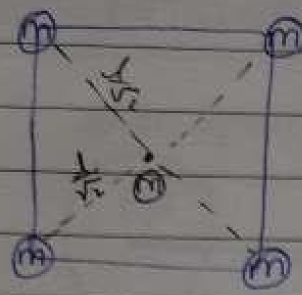
$$\Delta u \Rightarrow GMm\left(\frac{1}{R} - \frac{1}{R+h}\right)$$

$$\Delta u = GMm\left(\frac{\cancel{R}+h-\cancel{R}}{R(R+h)}\right)$$

$$\Delta u \Rightarrow \frac{GMm\,h}{R(R+h)}$$

$$\Delta u = \frac{gR^2 mh}{R(R+h)} = \frac{mgh\cancel{R^2}}{\cancel{R^2}\left(1+\frac{h}{R}\right)} = \frac{mgh}{1+\frac{h}{R}}$$

सिर्फ work पूछा हो तो हम W_{ext} force $= \Delta U$ निकालेंगे



$U_i = U_0$

$$= U_f - U_i$$

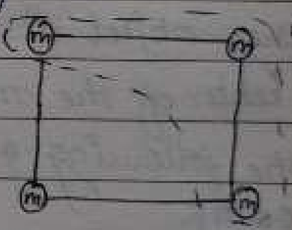
$$= \left[-\frac{Gm^2}{\frac{l}{\sqrt{2}}} \cdot 5 + \cancel{U_0}^{0}\right] - \cancel{U_0}^{0}$$

$$\Rightarrow -\frac{4\sqrt{2}Gm^2}{l}$$

U_i = Energy of the system

Q Four particles A, B, C, D each of mass m are kept at the corners of a square of side L. Now the particle D is taken to infinity by an external agent keeping the other particles fixed at their respective position. The work done by the gravitational force acting on the particle during its movement is.

Ans Here we have to find work done by gravitational force



$W_{gravitational\ force} = -\Delta U$

$$\Rightarrow -(U_f - U_i)$$

$$\Rightarrow U_i - U_f$$

$$\Rightarrow \cancel{U_0}^{0} - \frac{2Gm^2}{l} - \frac{Gm^2}{\sqrt{2}l} - \cancel{U_0}^{0}$$

$$\Rightarrow -\frac{Gm^2}{l}\left(2 + \frac{1}{\sqrt{2}}\right)$$

$$\Rightarrow -\frac{Gm^2}{l}\left(\frac{2\sqrt{2}+1}{\sqrt{2}}\right) \quad \text{Ans}$$

$$\boxed{\Delta U = \frac{mgh}{1+\frac{h}{R}}}$$

If $h <<<<< R$

Ex $h = 10m$ (very small)

$\Delta U = mgh$

Q An object is taken to height $2R$ above the surface of earth, the increase in potential energy is [R is radius of Earth)

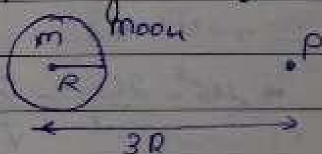
Ans $\Delta U = \frac{mgh}{1+\frac{2R}{R}} = \frac{mgh}{3} \Rightarrow \frac{mg2R}{3} = \frac{2mgR}{3}$

Q The change in potential energy when a body of mass m is raised to height nR from the earth's surface is.

Ans $\Delta U = \frac{mgh}{1+\frac{h}{R}} = \frac{mg\,nR}{1+n} = mgR\left(\frac{n}{1+n}\right)$

Q A stationary object is released from a point P at a distance 3R from the centre of the moon which has radius R and mass M. Which of the following gives the speed of the object on hitting the moon?

Ans By C.O.M.E.



$[K.E_i + U_i] = [K.E_f + U_f]$

$0 - \frac{GM\cancel{m}}{3R} = \frac{1}{2}\cancel{m}v^2 - \frac{GM\cancel{m}}{R}$

$\Rightarrow \frac{-GM}{3R} + \frac{GM}{R} = \frac{v^2}{2}$

$\Rightarrow \frac{2GM}{3R} \times 2 = v^2$

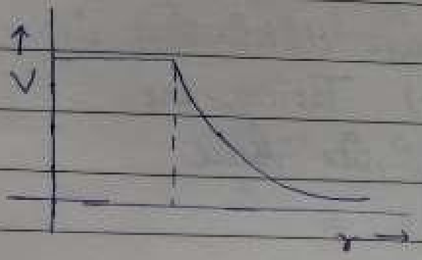
$v = \sqrt{\frac{4GM}{3R}}$ Ans

★

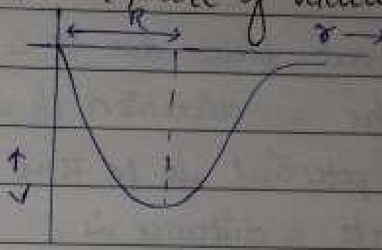
Q Find work done in bringing 5th mass m from infinity to the centre of the square.

Q which of the following curve expresses the variation of gravitational potential with distance for a hollow sphere of radius R.

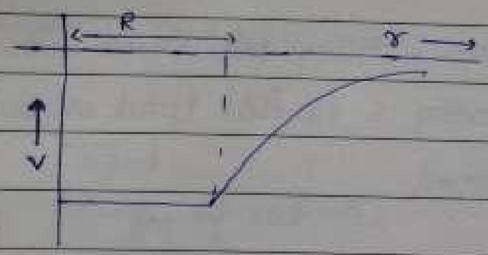
(A)



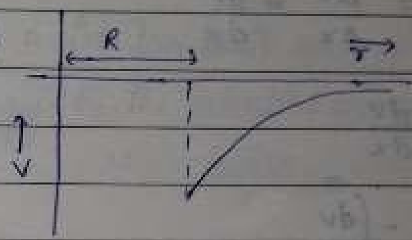
(B)



~~(C)~~



(D)



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Q Gravitational potential ~~density~~ difference b/w surface of a planet and a point situated at a height of 20 m above its surface is 2 J/kg. If Gravitational field is uniform, then the work done in taking a 5 kg body upto height 4 m above surface will be.

Ans → $\Delta V = 2\,J/kg$

to calcute work done $= -\Delta U$

$\Rightarrow \cancel{mgh}\ mgh$ → unknown for this planet

∴ let a mass be of 'm' kg

∴ $2\frac{J}{\cancel{kg}} \times \cancel{m}\,\cancel{kg} = \cancel{m}gh$

$2J = g \times 20$

$g = \frac{2}{20} = 0.1$

$mgh = 5 \times \frac{1}{10} \times 4 = 2J$ Ans

OR $I = \frac{\Delta V}{\Delta x} = \frac{2}{20} = 0.1 = g$

$mgh = 5 \times 0.1 \times 4 = 2J$ Ans

★

Q The magn of intensity of gravitational field at a point situated at a distance 8000 km from the centre of Earth is 6.0 N-kg. The magn of gravitational potential at the point in N-m/kg will be.

Ans $I = \frac{GM}{r^2}$ $\quad V = \frac{-GM}{r}$

$|V| = \frac{GM}{r} = 6 \times r = 6 \times 8000 \times 10^3 = 4.8 \times 10^7$ Nm/kg

$\frac{GM}{r} = 6 \times r$

Q If M_e is the mass of earth and M_m is the mass of moon ($M_e = 81 M_m$) The potential energy of an object of mass m situated at a distance R from the centre of earth and r from the centre of moon, will be.

Ans $U = \cancel{\frac{GM_e m}{}} - \frac{GM_e m}{R} - \frac{GM_m m}{r}$

$$\Rightarrow -\frac{G\,81 M_m m}{R} - \frac{GM_m m}{r}$$

$$\Rightarrow -GM_m m\left(\frac{81}{R} + \frac{1}{r}\right) \quad \text{Ans}$$

Q The gravitational potential energy is maxm at
→ Infinity.

Q A missile is launched with a velocity less than the escape velocity. Sum of its K.E and P.E is

a) Positive
~~b)~~ Negative
(c) May be negative or positive depending upon its initial velocity.
d) zero

→ ~~Initi~~ As total M.E is conserved ∴ Final M.E. = Initial M.E

Final M.E. = $\frac{-GMm}{R+h}$ → negative

Q The ratio of radii of two satellites is p and the ratio of their acc due to gravity is q. The ratio of their escape velocity will be

Ans $v_e = \sqrt{2g_1R_1}$

$$\frac{v_{e_1}}{v_{e_2}} = \frac{\sqrt{\cancel{2}g_1R_1}}{\sqrt{\cancel{2}g_2R_2}} = \sqrt{pq} \quad \text{Ans}$$

Q Escape velocity of a body from earth is 11.2 km/s. Escape velocity when thrown at an angle 45° from horizontal will be.

Ans It does not depend on angle of projection.

LIMITED EDITION

of the spheres is :-

Ans

$$-\frac{Gm}{r}\times 2 - \frac{Gm}{r}\times 2$$

$$\Rightarrow -\frac{4Gm}{r} \text{ Ans}$$

Q An artificial satellite moving in a circular orbit around the earth has a total (Kinetic + potential) energy E_0. Its potential energy is :-

Ans Total Energy of satellite $E_0 = -\frac{GMm}{2r}$ $U = -\frac{GMm}{r}$

$K.E. = \frac{GMm}{2r}$

Ans $\rightarrow 2E_0$

Q ★ A particle of mass m is moving in a horizontal circle of radius R under a centripetal force equal to $-\frac{A}{r^2}$ (A = constant). The total energy of the particle is (Potential energy at very large distance is zero)

Ans $\frac{\cancel{-}A}{r^2} = \frac{\cancel{-}mv^2}{r}$

$$\frac{1}{2}mv^2 = \cancel{-}\frac{A}{r}\times\frac{1}{2} = K.E. = \frac{A}{2R}$$

$$F = -\frac{dU}{dr}$$

$$T.E. = -\frac{A}{r} + \frac{A}{2R} = \frac{-A}{2R}$$

$$U = -\int F\,dr$$

$$U = -\int -\frac{A}{r^2}\,dr$$

$$U = \frac{-A}{r}$$

Q Potential energy of a 3 kg body at the surface of a planet is -54 J. then escape velocity is

Ans $V_e = \sqrt{\frac{2GM}{R}}$ $\qquad \cancel{-}\frac{GMm}{R} = \cancel{-}54$

$\frac{GM}{R} = 18$

$V_e = \sqrt{36}$

$V_e = 6$ m/s

LIMITED EDITION

OR $\quad I = \frac{\Delta V}{\Delta x} \quad = 6.0 \times 8 \times 10^6 m$

4) The Gravitational field due to a certain mass distribution is $E = \frac{K}{x^2}$ in the x - direction (k is a constant). Taking the Gravitational potential to be zero at infinity, its value corresponding to a distance is.

Ans $\quad E = \frac{K}{x^3} = \frac{\Delta V}{\Delta x} = \frac{dV}{dx}$

$$\frac{K}{x^3} = -\frac{dV}{dx}$$

$$\int \frac{K}{x^3} dx = -\int_{\infty}^{x} dV$$

$$\frac{-K}{2x^2} = \cancel{-(V_f - V_i)} \; -(V_f - V_i^{\,0})$$

$$\frac{+K}{2x^2} = +V_f$$

$$V_f = \frac{K}{2x^2} \quad \text{Ans}$$

Q The Gravitational potential energy of a body at a dist^n r from the centre of the earth is U. The force at that point is.

Ans $\quad F = -\frac{dU}{dR} = \frac{U}{r}$ (magnitude)

Q A projectile of mass m is thrown vertically up with an initial velocity n from the surface of earth (mass of earth = M). If it comes to rest at a height h, the change in its potential energy is.

Ans $\quad \Delta U = \frac{mgh}{1+\frac{h}{R}} = \frac{m\frac{GM}{R^2}h}{\frac{R+h}{R}} = \frac{GMmh}{R(R+h)}$

Q Two small and heavy spheres, each of mass M are placed a distance r apart on a horizontal surface. The Gravitational ~~constant~~ potential at the mid-point on the line joining the centre

Q The height vertically above the earth's surface at which the acceleration due to gravity g becomes 1% of its value at the surface is.

Ans
$$\frac{g}{100} = \frac{gR^2}{(R+h)^2}$$

$$\frac{R+h}{10} = R$$

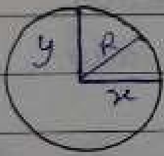
$$R+h = 10R$$

$$h = 9R \quad \text{Ans}$$

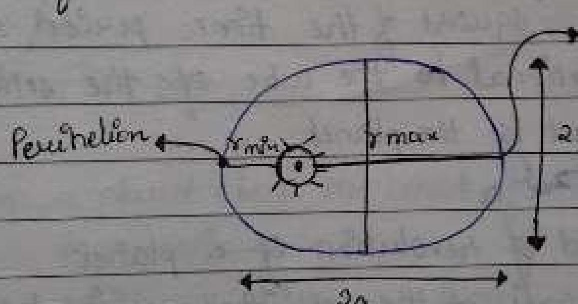
KEPLERS LAW

→ There are three basic laws of ~~potential~~ planetary motion given by Kepler can be stated as follows

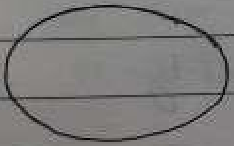
1. Law of orbits :- All planets move in elliptical orbits, with the sun at the foci of the ellipse



R



b = semi minor radius



$$r_{max} + r_{min} = 2a$$

$$r_{max} = (1+e)a$$

a = semi major radius

$$r_{min} = (1-e)a$$

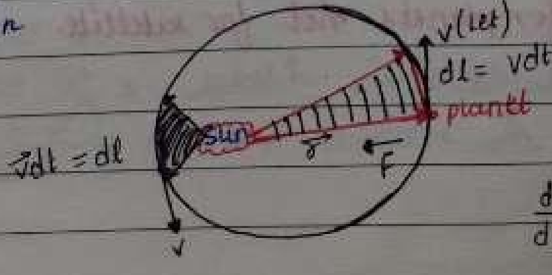
e = eccentricity

$$\frac{r_{max}}{r_{min}} = \frac{(1+e)\cancel{a}}{(1-e)\cancel{a}} = \frac{1+e}{1-e}$$

2. Law of Area: The line that joins any planet to the sun sweept out equal areas in equal intervals of time.

Areal velocity is constant

$$\frac{dA}{dt} = \text{const}^n$$



Area of triangle $= \frac{1}{2}\vec{r}\,vdt$

$$dA = \frac{1}{2}\vec{r}\,vdt$$

$$\frac{dA}{dt} = \frac{1}{2}\vec{r}\vec{v} = \text{constant}$$

$$\therefore \vec{r}\vec{v} = \text{constant}$$

$\frac{1}{2}\frac{mrv}{m}$ = constant

$$\boxed{\frac{L}{2m} = \text{constant}}$$

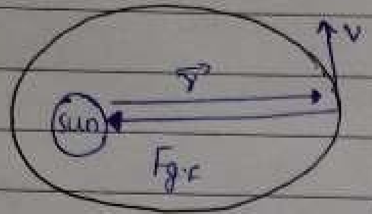
$\frac{dA}{dt}$ = constant

$\frac{1}{2}\vec{r}\times\vec{v}$ = constⁿ

rv = constant

$v \propto \frac{1}{r}$

$\frac{\vec{L}}{2m}$ = constant



torque due to gravitational force on planet about sun is = 0

$\tau = \frac{dL}{dt} = 0$ $\quad L$ = constⁿ

mvr = constⁿ

vr = constⁿ

Accⁿ of planet is always towards the sun → True.

3) Law of Periods : The square of the time period of revolution of a planet is proportional to the cube of the semi-major axis of the ellipse traced out by the planet

$$T^2 \propto a^3$$

T = Time period of revolution of a planet

a = Semi major axis of the elliptical orbit traced out by the planet

Q According to Kepler, planets move in.

a) Circular orbits around the sun

b) Elliptical orbits around the sun with sun at exact centre

c) Straight lines with constant velocity.

d) ✓ Elliptical orbits around the sun with sun at one of its foci.

Keplers law is for planets not for satellite.

Q The minimum and maximum distances of a planet revolving around sun are r and R. If the minimum speed of planet on its trajectory is V_0 its max^m speed will be.

Ans $v \propto \frac{1}{r}$

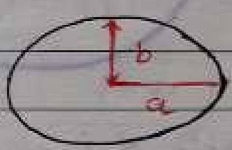
$$r V_{max} = R V_{min}$$

$$r V_{max} = R V_0$$

$$V_{max} = \frac{R V_0}{r}$$ Ans

Q A planet of mass m moves around the sun of mass M in an elliptical orbit. The max^m and min^m distances of the planet from the sun are r_1 and r_2 respectively. The time period of the planet is proportional to.

Ans $T^2 \propto a^3$



$T \propto a^{3/2}$

$2a = r_1 + r_2$

$a = r_1 + r_2/2$

$\therefore T \propto (r_1 + r_2)^{3/2}$ Ans

Q The torque on a planet about the centre of sun is.

a) Zero b) Negative

c) Positive d) Depend on mass of planet.

Q During motion of a planet from perihelion to aphelion the work done by gravitational force of sun on it is.

Ans Negative

$W = \Delta K.E$

From perihelion to aphelion speed $\downarrow$ $\therefore$ K.E $\downarrow$

$\therefore W = \Delta K.E = -ve$

$W \downarrow$

Q The timeperiod of a satellite in a circular orbit of radius R is T. The timeperiod of another satellite in a circular orbit of radius 4R is

LIMITED EDITION

$T^2 \propto R^3$

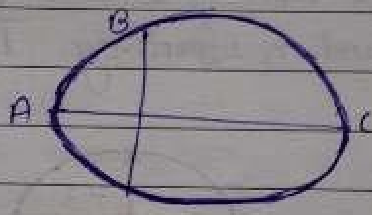
$T'^2 \propto 64R^3$

$\Rightarrow \frac{T^2}{T'^2} = \frac{R^3}{64\cancel{R^3}}$

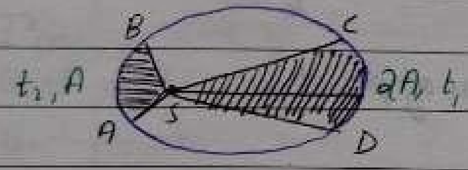
$T' = 8T$ Ans

Q The Kinetic energies of a planet in an elliptical orbit about the sun, at the positions A, B and C are K_A, K_B and K_C respectively. AC is the major axis and SB is perpendicular to AC at the position of the sun S as shown in the figure. Then

Ans $K_A > K_B > K_C$



Q The figure shows elliptical orbit of a planet m about the sun S. The shaded area SCD is twice the shaded area SAB. If t_1 is the time for the planet to move from C to D and t_2 is the time to move from A to B then find relation b/w t_1 and t_2



$t_1 = 2t_2$

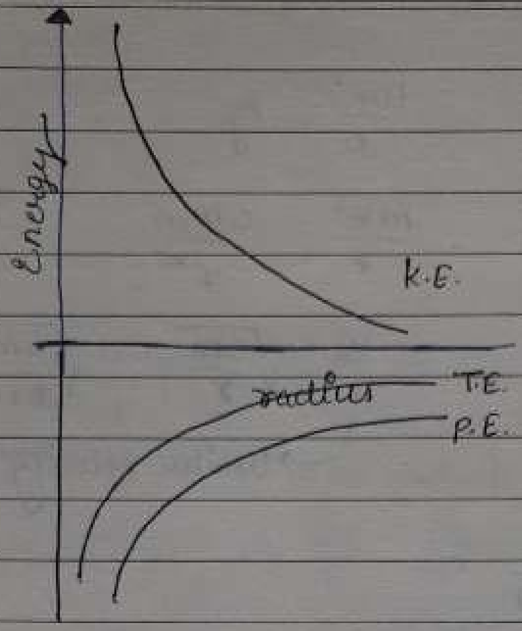
SATELLITE (ORBITAL VELOCITY)

A satellite of mass m. revolving around earth with a speed of v in a circular orbit of radius $(R_e + h)$ where R_e is the radius of the earth

$U = 2\,T.E.$

$K.E. = +T.E.$

$|U| = 2\,K.E.$



SATELLITE ON CIRCULAR PATH

K.E. = constant

P.E. = constant

M.E. = constant

L = constant

$\tau = 0$

$\vec{p}$ = variable

$\vec{v}$ = variable

Work = 0 (K.E. = constn)

↳ or no displacement

Speed constant

Time period of satellite

$$T = \frac{2\pi R}{V_0} \quad (h <<< R)$$

$$T = \frac{2\pi R}{\sqrt{\frac{GM}{R}}}$$

sq both side

$$T^2 = \frac{4\pi^2 R^2}{\frac{GM}{R}} = \frac{4\pi^2 R^3}{GM}$$

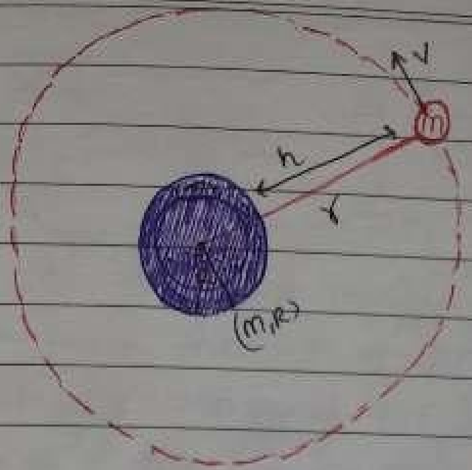
$\boxed{T^2 \propto R^3}$ → For satellite

BINDING ENERGY

ENERGY given to the body so that it becomes free.

Bounded Energy = $+|T.E.|$

Free energy = $T.E. \to 0$



$$\frac{mv^2}{r} = F_g$$

$$\frac{\cancel{m}v^2}{\cancel{r}} = \frac{GM\cancel{m}}{r^{\cancel{2}}}$$

$$v_o = \sqrt{\frac{GM}{r}} = \sqrt{\frac{GM}{R+h}}$$

v_o → Orbital velocity of satellite

If $(h << R)$

$$v_o = \sqrt{\frac{GM}{R+h}} = \sqrt{\frac{GM}{R}}$$

$$v_e = \sqrt{\frac{2GM}{R}} \qquad \frac{v_e}{v_o} = \sqrt{2}$$

$$\boxed{v_e = \sqrt{2}\,v_o} \quad \underline{\text{Ans}}$$

$$v_e = 1.41\, v_o$$

$$v_e = 141.4\%\, v_o$$

Q When speed of a satellite is increased by x- percentage, it will escape from its orbit, where the value of x is.

→ 41.4% Ans

Q A small satellite is revolving near earth's surface. Its orbital velocity will be nearly.

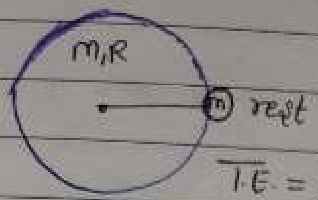
Ans $v_o = \sqrt{\frac{GM}{R}} = \frac{v_e}{\sqrt{2}} = \frac{11.2}{\sqrt{2}} = 8\ km/s$

K.E. of satellite

$$K.E. = \frac{1}{2}mv_o^2 \Rightarrow \left(v_o = \sqrt{\frac{GM}{R}}\right)$$

$$\boxed{T.E. = -\frac{GMm}{2R}}$$

$$\boxed{K.E \Rightarrow \frac{GMm}{2R}} \qquad \boxed{P.E. = -\frac{GMm}{R}}$$



$B.E. = \left|\frac{-GMm}{R}\right| = \frac{GMm}{R}$

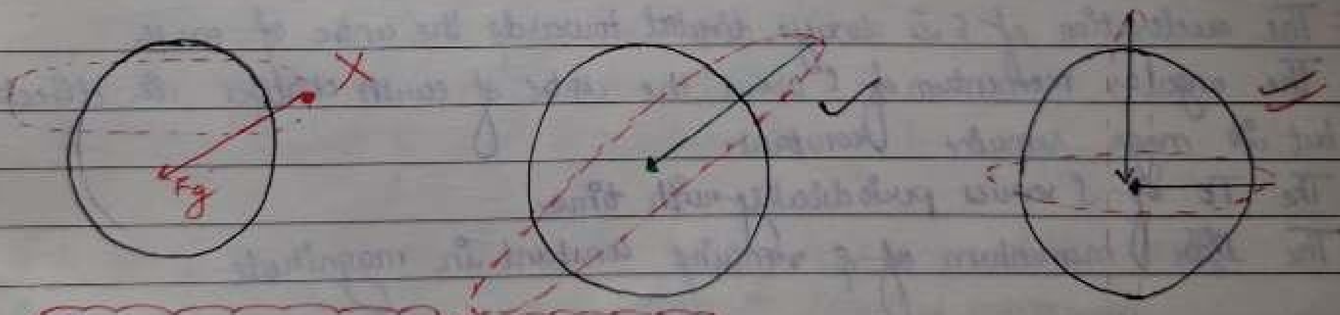
$T.E. = \cancel{K.E}^{0} + P.E$

It is the minimum energy given to the satellite so that it escapes away from the G.F. of the planet around which it is revolving. The binding energy of the Earth satellite is $E_B = \frac{GMm}{2r}$ where r is radius of the circular orbit.

1) The binding energy $E_B = \frac{GMm}{2r}$ is the negative of the total M.E., $TE = -\frac{GMm}{2r}$

2) The binding energy of a particle of mass m placed on the earth is $E_B = \frac{GMm}{R}$ where R is the radius of earth.

3) The binding energy of the body at rest on the Earth's surface is twice the binding energy of a satellite of same mass around the Earth very near to its surface (i.e. $r=R$)



GEOSTATIONARY SATELLITE

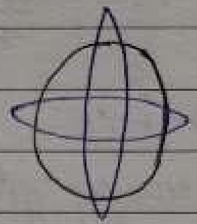
- → $h = 6R$ from surface $= 36000$ Km
- → $h = 7R$ from centre of earth (42000 Km)
- → $T = 24$ hr
- → $v = 3.1$ km/hr
- → Equatorial plane
- → west to east

- → 1 satellite cover → $\frac{1}{3}$rd Area of Earth
- → Communication, Parking
- → Relay satellite
- → Angular speed, time point; direction of rotation same as Earth
- → Remains at same position of earth.

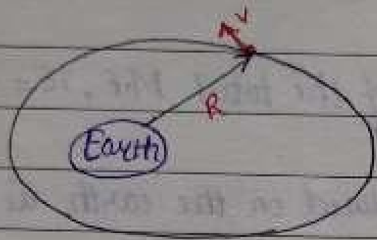
LIMITED EDITION

Polar Satellite

→ About pole
→ time period = 100 min



Satellite on Elliptical Path



- distance of satellite from planet → variable
- rv = constant
- Speed → variable
- K.E → variable
- P.E. → variable
- T.E = constant
- Linear momentum → variable

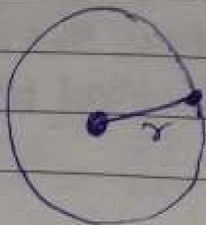
Q A satellite S is moving in an elliptical orbit around the Earth. The mass of the satellite is very small compared to the mass of the earth. Then

a) The acceleration of S is always directed towards the centre of earth
b) The angular momentum of S about the centre of earth changes in direction but its magn remains constant
c) The T.E. of S varies periodically with time.
d) The linear momentum of S remains constant in magnitude.

Q Kepler's third law states that the period of revolution (T) of a planet around the sun, is proportional to the third power of average distance r b/w sun and planet, i.e. $T^2 = kr^3$ and k is constant. If the masses of sun and planet are M and m respectively then as per newton's law of ~~gra~~ gravitational force of attraction b/w them is $F = \frac{GMm}{r^2}$, here G is gravitational constant. The relation b/w G and k is described as

LIMITED EDITION

Ans



$\frac{GMm}{r^2} = m\omega^2 r$

$T^2 GM = 4\pi^2 r^3$

$T^2 GM = 4\pi^2 r^3$

$K r^3 GM = 4\pi^2 r^3$

$GMK = 4\pi^2$

Q The additional K.E. to be provided to a satellite of mass m

★ revolving around a planet of mass M, at transfer it from a circular orbit of radius R_1 to another of Radius R_2 $(R_2 > R_1)$ is.

Ans $T_f - TE_i$

$\Rightarrow -\frac{GMm}{2R_2} + \frac{GMm}{2R_1}$

$\Rightarrow \frac{GMm}{2}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

Q Two satellites of earth S_1 and S_2 are moving in the same orbit. The masses of S_1 is four times the mass of S_1. Which one of the following statement is true?

a) The Potential energy of Earth and satellite in the two cases are equal

b) S_1 and S_2 are moving with the same speed. $\left(V_0 = \sqrt{\frac{GM}{r}}\right)$

c) The K.E. of the two satellites are equal.

d) The Time period of S_1 is four time that of S_2

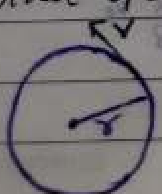
Q If a ~~speed~~ satellite of mass 400 kg revolves around the earth in an orbit with speed 200m/s then its potential energy is.

Ans $|P.E| = 2K.E.$

$|P.E| = \cancel{2} \times \frac{1}{\cancel{2}} \times 400 \times 200 \times 200 = 16 \times 10^6 \Rightarrow -16\ MJ$ Ans

Q An artificial satellite revolves around a planet for which gravitational force (F) varies with distance r from its centre as $F \propto r^2$. If V_0 its orbital speed, then

Ans



$\frac{mv^2}{r} = GMm r^2$ $\quad v \propto r^{3/2}$

Ques Assume that the force of gravitation $F \propto \frac{1}{r^n}$. The show that the orbital speed in a circular orbit of radius r is proportional to $\frac{1}{r^{(n-1)/2}}$ while its time period T is proportional to $r^{(n+1)/2}$

Ans $\frac{\cancel{m} v_o^2}{r} = \frac{GM\cancel{m}}{r^n}$ $\qquad v_o \propto \frac{1}{r^{(n-1)/2}}$

$v_o^2 \propto \frac{1}{r^{n-1}}$ $\qquad \cancel{m}\omega^2 r = \frac{GM\cancel{m}}{r^n}$

$v_o^2 \propto \frac{1}{r^{(n-1)}}$ $\qquad \frac{4\pi^2}{T^2} r = \frac{GM}{r^n}$

$r \cdot r^n = T^2$

$r^{(n+1)/2} = T$

Q If Gravitational field intensity is E at a distance R/2 outside from the surface of thin spherical shell of radius R, the gravitational field intensity at distance R/2 from its centre is.

Ans → Zero

Q If accn due to gravity at distance d (d<R) from the centre of earth is β, then its value at distance d above the surface of earth will be [where R is radius of earth]

Ans $\frac{B = \frac{GMd}{R^3}}{x = \frac{gR^2}{(R+h)^2}} \Rightarrow \frac{\beta = \frac{GMd}{R^3}}{x = \frac{gR^2}{(R+d)^2}} = \frac{\beta = \frac{GMd}{R^3}}{x = \frac{GM}{(R+d)^2}}$ $\qquad \cancel{\frac{\beta}{x} = \frac{Md}{R^3(R+d)^2}}$

$\Rightarrow \frac{\beta}{x} = \frac{(R+d)^2 d}{R^3}$

$x = \frac{\beta R^3}{(R+d)^2 d}$ Ans

Q If the gravitational potential on the surface of earth is V_o then potential at a point at height half of the radius of earth is

Ans $V' = -\frac{GM}{R+h} = \frac{-GM}{\frac{3R}{2}} = \frac{-2GM}{3R} = \frac{2}{3}V_o = \frac{2}{3}V_o$

Q Gravitational potential in a region is given by $V = -(x+y+z)$ J/Kg. Find the gravitational intensity at (2,2,2)m.

Ans $I = -\frac{\partial V}{\partial r} = -\left(\frac{dV}{dx}\hat{i} + \frac{dV}{dy}\hat{j} + \frac{dV}{dz}\hat{k}\right)$

$\Rightarrow -\left(\frac{d\, x+y+z}{dx}\hat{i} + \frac{d\, x+y+z}{dy}\hat{j} + \frac{d\, x+y+z}{dz}\hat{k}\right)$

$\Rightarrow -\left(1\hat{i} + \hat{j} + \hat{k}\right)$

$\Rightarrow \hat{i} + \hat{j} + \hat{k}$ N/kg Ans

Q At what height from the surface of earth the gravitation potential and the value of g are -5.4×10^7 J/Kg and 6.0 m/s² respectively? Take the radius of earth as 6400 km.

Ans $V = -\frac{GM}{R+h}$ $g = \frac{GM}{(R+h)^2}$

$GM = V(R+h)$

$g = \frac{V}{R+h}$

$6(64 \times 10^5 m + h) = 5.4 \times 10^7$ 0.9

$h = 9 \times 10^6 - 6.4 \times 10^6$

$h = 2.6 \times 10^6$ m

$h = 2600 \times 10^3$ m $= 2600$ km

Q A particle of mass 'm' is kept at rest at a height 3R from the surface of earth, where 'R' is radius of earth and 'M' is mass of earth. The min^m speed with which it should be projected, so that it does not return back, is.

Ans $V_{escape} = \sqrt{\frac{2GM}{R+h}} = \sqrt{\frac{2GM}{4R}} = \sqrt{\frac{GM}{2R}}$ Ans

Q The gravitational force b/w two objects were proportional to 1/R where R is the distance b/w them, then a particle in a circular path would have its orbital speed v, is proportional to

Ans $\frac{\cancel{m}v^2}{\cancel{R}} = \frac{GM\cancel{m}}{\cancel{R}}$ $v = \sqrt{GM}$ independent of R

LIMITED EDITION

Q Two astronauts are floating in gravitational free space after having lost contact with their spaceship. The two will

a) move towards each other.

b) move away from each other.

c) will become stationary

d) keep floating at the same distance b/w them.

Ans gravity free → only defined for planet
force due to planet is zero. $g = 0$.

Gravitational free → Phenomenon of gravitational force is absent $G = 0$.

Q If the mass of the sun were ten times smaller and the universal gravitational constant were 10 times larger in magn, which of the following is not correct.

a) Raindrops will fall faster.

b) walking on the ground would become more difficult

c) Time period of a simple pendulum on the Earth would decrease.

d) g on the Earth will not change

$$g = \frac{GM_e}{R^2}$$ $$T = 2\pi\sqrt{\frac{l}{g}}$$

Q A spherical planet has a mass M_p and diameter D_p. A particle of mass m falling freely near the surface of this planet will experience an accn due to gravity equal to.

Ans $g = \frac{4GM_p}{D_p^2}$

Q The radius of earth is about 6400 km and that of mars is 3200 km. The mass of the earth is about 10 times mass of mars. An object weighs 200 on the surface of Earth. Its weight on the surface of earth. Its wt. on the surface of mars will be.

Ans $R_e = 2R_m$

$M_e = 10M_m$

$g = \frac{GM_m}{R_m^2} = \frac{G M_e \times 4}{10\, R_e^2} = \frac{4}{10}$ times $= \frac{4}{10} \times 200 = 80N$

Q A body weighs 72 N on the surface of the earth. What is the gravitational force on it, at a height equal to the radius of the earth. ?

Ans $g_e = \frac{gR^2}{9R^2} \times 4 \Rightarrow \frac{4}{9}g$

$mg' = \frac{4}{9}mg = \frac{4}{9} \times 72 = 32N$

Q A body weighs 200 N on the surface of the earth. How much will it weigh halfway down to the centre of earth?

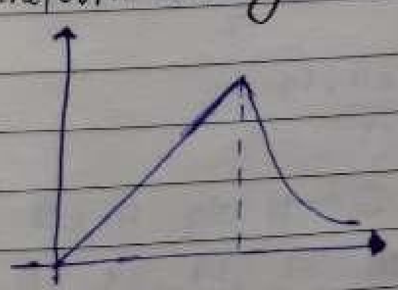
Ans $g_e = g\left(1 - \frac{R}{2R}\right) = \frac{g}{2}$

$mg' = mg \times \frac{1}{2} = 100N$

Q The height at which the wt. of a body becomes $\frac{1}{16}$th, its wt. on the surface of earth

Ans $\frac{1}{16} = \frac{R^2}{(R+h)^2} \Rightarrow \frac{1}{4} = \frac{R}{R+h} \Rightarrow R+h = 4R \Rightarrow h = 3R$ Ans

Q The dependence of accn due to gravity g on the dist^n r from the centre of earth, assumed to be a sphere of radius R and of uniform density is as shown in figures.



Q The work done to raise a mass m from the surface of the earth to a height h, which is equal to the radius of the earth, is

Ans $W = \Delta U = \frac{mgR}{1+\frac{h}{R}} = \frac{mgR}{1+\frac{R}{R}} = \frac{mgR}{2}$

Q Infinite no. of bodies, each of mass 2 kg are situated on x axis at distance 1m, 2m, 4m, 8m... respectively from the origin. The resulting gravitational potential due to this system at the origin will be

Ans $-2G - \frac{2G}{2} - \frac{2G}{4} - \frac{2G}{8}$

$\Rightarrow -2G\left(1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}\right)$

$\Rightarrow -2G\left(\frac{1}{1-\frac{1}{2}}\right) = -2G\left(\frac{2}{1}\right) = -4G$

Q The ratio of escape velocity at earth (v_e) to the escape velocity at a planet (v_p) whose radius and mean density are twice as that of earth is.

Ans $V = R\sqrt{\rho}$ $\left(V = \sqrt{\frac{8}{3}\pi \rho G R^2}\right)$

$\frac{v_e}{v_p} = \frac{R\sqrt{\rho}}{R\,2\sqrt{2\rho}} = \frac{v_e}{v_p} = \frac{\cancel{R}\cancel{\sqrt{\rho}}}{2\sqrt{2}\sqrt{\rho}\cancel{R}}$

$\frac{v_e}{v_p} = 1 : 2\sqrt{2}$ Ans

Q A black hole is an object whose gravitational field is so strong that even light cannot escape from it. To what approximate radius would earth (mass = 5.98×10^{24}) have to be compressed to be a black hole

Ans $v_e = c$

$\sqrt{\frac{2Gm}{R}} = c$

$\frac{2Gm}{R} = c^2$

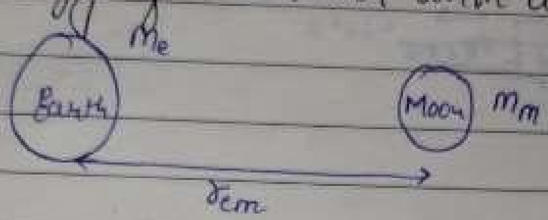
$R = \frac{2Gm}{c^2} = \frac{2 \times 6.67 \times 10^{-11} \times 6 \times 10^{24}}{9 \times 10^{16}} = 8.86 \times 10^{-3}m \approx 10^{-2}m$

Q A satellite of mass m is orbiting the earth (of Radius R) at a height h from its surface. The total energy of the satellite in terms of g_0 the value of accn due to gravity at the earth's surface is

Ans $T.E. = -\frac{GMm}{2(R+h)} \Rightarrow -\frac{g_0 R^2 m}{2(R+h)}$ Ans

Q Binding energy of moon and earth is

Ans



$B.E. = + T.E.$

$\Rightarrow \frac{GM_e M_m}{2 r_{em}}$ Ans

Q Kepler's second law is a consequence of

a) Conservation of K.E.

b) Conservation of linear momentum.

c) Conservation of angular momentum.

d) Conservation of speed.

Q If a graph is plotted b/w T^2 and r^3 for a planet then its slope will be

Ans $\cancel{m}\omega^2 R = \frac{GM\cancel{m}}{R^2}$

$\frac{4\pi^2}{T^2} R^3 = GM$

$R^3 \times \frac{4\pi^2}{T^2} = GM$

$T^2 = \frac{4\pi^2}{GM} R^3$

$y = mx + c$, $c \to 0$

$m = \text{slope} = \frac{4\pi^2}{GM}$ Ans

Q A planet is moving in an elliptical orbit. If T, U, E, L are its K.E, P.E, T.E. and magn of angular momentum respectively, then which of the following statement is true.

A) T is conserved

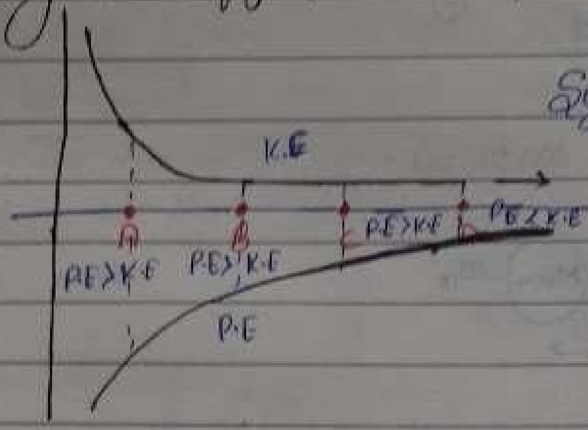
B) U is conserved

C) E is always negative

D) L is conserved but the dirn of vector $\vec{L}$ will continuously change.

LIMITED EDITION

Q Potential energy and Kinetic energy of a two particle system under imaginary force field are shown by curves KE and PE respectively in figure. This system is bound at.



System is bound when T.E < 0

Answer → A, B, C

Q Two artificial satellites A and B are at a distance r_A and r_B above the earth's surface. If the radius of earth is R then the ratio of their speed will be.

Ans $V_{orbital} = \sqrt{\frac{GM}{r_B + R}}$

$$V \propto \frac{1}{\sqrt{r+R}} \quad \therefore \frac{V_A}{V_B} = \frac{\sqrt{r_B+R}}{\sqrt{r_A+R}} = \left(\frac{r_B+R}{r_A+R}\right)^{1/2} \text{ Ans}$$

Q The average radii of orbits of mercury and earth around the sun are 6×10^7 km and 1.5×10^8 km respectively. The ratio of their orbital speeds will be.

Ans $\frac{V_{om}}{V_{oe}} = \frac{\sqrt{r_e}}{\sqrt{r_m}} = \sqrt{\frac{15 \times 10^7}{6 \times 10^7}} = \sqrt{5} : \sqrt{2}$ Ans

Q A body is dropped by a satellite in its geostationary orbit.

(A) It will burn on entering in to the atm

~~(B)~~ it will remain in the same place w.r.t. the earth

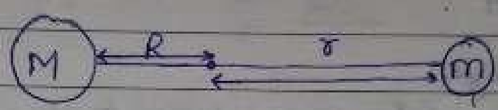
(C) it will reach the earth in 24 hours

(D) it will perform certain motion.

Q If two bodies of mass M and m are revolving around the com of the system in circular orbit of radii R and r respectively due to mutual interaction. Which of the following formula is applicable:

(a) $\frac{GMm}{(R+r)^2} = m\omega^2 R$ (b) $\frac{GMm}{R^2} = \cancel{m\omega^2}\, m\omega^2 r$

(c) $\frac{GMm}{r^2} = m\omega^2 R$ (D) $\frac{GMm}{R^2+r^2} = m\omega^2 Rr$



$F_c = F_G$

$\underset{\downarrow}{m\omega^2 r} = \frac{GMm}{(R+r)^2}$

As it is revolving around c.o.m.

Q Two satellites of same mass m are revolving around the earth (mass M) in the same orbit of radius r. Rotational directions of the two are opposite therefore, they can collide. Total M.E. of the system is.

Ans $\Rightarrow TE_1 + TE_2$

$\Rightarrow -\frac{GMm}{2r} - \frac{GMm}{2r} = -\frac{\cancel{2}GMm}{\cancel{2}r} \Rightarrow -\frac{GMm}{r}$ Ans

Q Two artificial satellites of masses m_1 and m_2 are moving with speeds v_1 and v_2 in orbits of radii r_1 and r_2 respectively. If $r_1 > r_2$ then which of the following statement is true.

a) $v_1 = v_2$ (b) $v_1 > v_2$

(c) $v_1 < v_2$ (d) $\frac{v_1}{r_1} = \frac{v_2}{r_2}$

Ans $v = \sqrt{\frac{GM}{r}} \Rightarrow v \propto \frac{1}{\sqrt{r}}$ $\therefore v_1 < v_2$ $(\because r_1 > r_2)$

LIMITED EDITION

Q Orbital radius of a satellite S of earth is four times that of a communication satellite C. Period of revolution of S is.

Ans $T = 1 \text{day}$

$$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{r_1}{r_2}\right)^3 \qquad r_2 = 4r_1$$

$\Rightarrow T_1 = 1$

$$\Rightarrow \left(\frac{1}{T_2}\right)^2 = \left(\frac{\cancel{r_1}}{4\cancel{r_1}}\right)^3$$

$$\Rightarrow \left(\frac{1}{T_2}\right)^2 = \frac{1}{64}$$

$T_2 = 8$ days.

Q If a satellite is revolving very close to the surface of earth, then its orbital velocity does not depend upon

Ans ~~a)~~ Mass of satellite (b) Mass of earth
c) Radius of Earth (d) Orbital radius

$$v = \sqrt{\frac{GM}{r}}$$

Q A body of mass m is kept at a small height h above the ground. If the radius of the earth is R and its mass is M, the potential energy of the body and earth system (with $h = \infty$ being the reference point) is

Ans
$$U_h = \frac{-GMm}{R+h}$$

$$U_h = -\frac{GMm}{R\left(1+\frac{h}{R}\right)}$$

$$U_h = -\frac{GMm}{R}\left(1+\frac{h}{R}\right)^{-1}$$

$$U_h = \frac{-GMm}{R}\left(1-\frac{h}{R}\right). \qquad \left((1+x)^{-1} \text{ if } x <<< 1 \text{ then } (1-x)\right)$$

$$U_h = -\frac{GMm}{R} + \frac{GMmh}{R^2}$$

$$U_h = -\frac{GMm}{R} + \frac{gR^2 mh}{R^2} \Rightarrow \frac{-GMm}{R} + mgh$$

LIMITED EDITION

Q Suppose the gravitational force varies inversely as the n^{th} power of distance. Then the time period of a planet in circular orbit of radius r around the sun will be proportional to.

Ans $\frac{G\cancel{m}M}{r^n} = \frac{\cancel{m}v^2}{r}$

$v = \sqrt{\frac{GM}{r^{n-1}}}$

$T = \frac{2\pi r}{v}$

$T = \frac{2\pi r}{\sqrt{GM}/r^{(n-1)/2}}$

$T = \frac{2\pi r\, r^{(n-1)/2}}{\sqrt{GM}}$

$T = 2\pi r^{(n-1)/2+1} / \sqrt{GM}$

$T \propto r^{(n+1)\frac{1}{2}}$ Ans

Q The orbital angular momentum of a satellite revolving at a distance r from the centre is L. If the distance is increased to $16r$, then the new angular momentum will be.

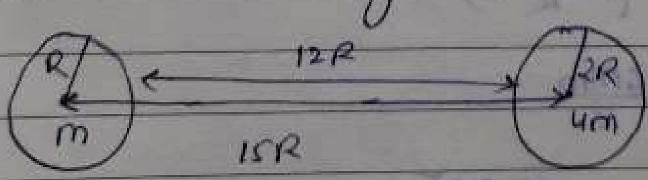
Ans $L = mvr = m\sqrt{\frac{GM}{r}}\, r = m\sqrt{GMr}$

$L \propto \sqrt{r}$

$\frac{L_1}{L_2} = \frac{\sqrt{r_1}}{\sqrt{r_2}} \Rightarrow \frac{L}{L_2} = \frac{\sqrt{\cancel{r}}}{\sqrt{16\cancel{r}}} = \frac{1}{4} \Rightarrow L_2 = 4L$ Ans

Q Two spherical masses m and $4m$ and radii R and $2R$ respectively are released in free space with initial separation b/w their centres equal to $15R$. If they attract each other due to G.F. only, then the dist^n covered by smaller sphere just before collision will be.

Ans



Dist^n b/w them = 12R

$s = ut + \frac{1}{2}at^2$

$\Rightarrow x = 0 + \frac{1}{\cancel{2}} \times a_m \cancel{t^2}$

$x - 12 = 0 + \frac{1}{\cancel{2}} a_{4m} \cancel{t^2}$

$F = \frac{G \times m \times 4m}{15R}$

$a_m = \frac{4m/\cancel{4}}{1\cancel{5}R}$ $\Rightarrow \frac{a_m}{a_{4m}} = 4$

$a_{4m} = \frac{m/\cancel{4}}{\cancel{5}R}$

$\Rightarrow \frac{x}{12-x} = 4$; $x = -4x + 48$

$5x = 48$

$x = 9.6R$

LIMITED EDITION

Q The gravitational potential energy of a body of mass m at a distance r from the centre of the earth is U. What is the weight of the body at this distance

Ans $U = \frac{-GMm}{r}$ $\frac{GM}{r} = \frac{U}{m}$.

$E = \frac{-GM}{r^2}$

$E = \frac{GM}{r \times r} \Rightarrow \frac{U}{mr}$

weight $= mg = mE = \frac{U}{r}$ Ans

Q The magnitude of gravitational potential energy of a body at a distance r from the centre of earth is U. Its weight at a distance 2r from the centre of earth is.

Ans $U = \frac{-GMm}{r}$ $E = \frac{GM}{r^2}$ $w = mg$

$\Rightarrow Em = \frac{U}{4r}$ Ans

$\frac{GM}{r} = \frac{U}{m}$ $E = \frac{GM}{4r^2}$

$E = \frac{U}{4mr}$

Q What is the energy required to launch m kg satellite from earth's surface in a circular orbit at an altitude of 7R?

Ans Energy required = T.E. at height 7R − T.E. at the surface

$\Rightarrow \frac{-GMm}{2(8R)} - \left(-\frac{GMm}{R}\right)$

$\Rightarrow \frac{-GMm}{16R} + \frac{GMm}{R}$

$\Rightarrow \frac{15GMm}{16R} \Rightarrow \frac{15\,gR^2m}{16R} = \frac{15}{16}mgR$

Q Energy of the satellite in circular orbit is E_0. The energy required to move a satellite to a circular orbit of 3 times the radius of the initial orbit is

Ans $E_0 = T.E. = -\frac{GMm}{2r}$

$$E' = -\frac{GMm}{6r} = \frac{E_0}{3}$$

$$E_0 - \frac{E_0}{3} \Rightarrow \frac{2E_0}{E}$$

Q A planet of mass m revolves in an elliptical orbit around the sun so that its maximum and minimum distance from the sun are equal to r_a and r_p respectively. The angular momentum of this planet relative to the sun is

Ans By law of conservation of angular momentum

$$m v_p r_p = m v_a r_a$$

And also $v_p r_p = v_a r_a$

By conservation of energy

$$\Rightarrow -\frac{GM\cancel{m}}{r_p} + \frac{1}{2}\cancel{m} v_p^2 = -\frac{GM\cancel{m}}{r_a} + \frac{1}{2}\cancel{m} v_a^2$$

$$\Rightarrow \frac{GM}{r_a} - \frac{GM}{r_p} = \frac{v_a^2 - v_p^2}{2}$$

$$\Rightarrow v_a = \frac{v_p r_p}{r_a}$$

$$\Rightarrow \frac{GMr_p - GMr_a}{\cancel{r_a} r_p} = \frac{v_p^2 r_p^2 - v_p^2 r_a^2}{2 r_a^2}$$

$$\Rightarrow \frac{GM(\cancel{r_p - r_a})}{r_p} = \frac{v_p^2 (r_p + r_a)(\cancel{r_p - r_a})}{2 r_a}$$

$$\Rightarrow \sqrt{\frac{2GMr_a}{r_p(r_p + r_a)}} = v_p$$

$$L = m v_p r_p \Rightarrow m\sqrt{\frac{2GM r_a r_p^2}{\cancel{r_p}(r_p + r_a)}} = m\sqrt{\frac{2GM r_a r_p}{(r_p + r_a)}}$$

LIMITED EDITION

Q Two particles of mass m and M are initially at rest at infinite distance. Find their relative velocities of approach due to Gravitational attraction when d is their separation at any instant.

Ans By conservation of energy

$$\Rightarrow 0+0 = -\frac{GMm}{d} + \frac{1}{2}mv_1^2 + \frac{1}{2}Mv_2^2$$

$$\Rightarrow \frac{GMm}{d} = \frac{1}{2}(mv_1^2 + Mv_2^2)$$

By conservation of momentum

$$m_1v_1 = Mv_2$$

$$v_1 = \frac{Mv_2}{m}$$

$$\frac{2G\cancel{M}m}{d} = \cancel{m}\frac{M^{\cancel{2}}v_2^2}{m^{\cancel{2}}} + \cancel{M}v_2^2$$

$$\Rightarrow \frac{2Gm}{d} = \frac{Mv_2^2}{m} + v_2^2$$

$$\Rightarrow \cancel{2Gm}\ \frac{2Gm}{d} = \frac{Mv_2^2 + mv_2^2}{m}$$

$$\Rightarrow \frac{2Gm^2}{d} = v_2^2(M+m)$$

$$v_2 = \sqrt{\frac{2Gm^2}{(M+m)d}}$$

Similarly

$$v_1 = \sqrt{\frac{2GM^2}{(m+M)d}}$$

relative velocity $= v_1 + v_2 \quad ((v_1 - (-v_2))$

$$v_1 + v_2 = \sqrt{\frac{2GM^2}{(m+M)d}} + \sqrt{\frac{2Gm^2}{m+M}}$$

$$v_1 + v_2 = \sqrt{\frac{2G}{(m+M)d}}\left(\sqrt{M^2} + \sqrt{m^2}\right)$$

$$v_1 + v_2 = \sqrt{\frac{2G(M+m)^2}{(m+M)d}}$$

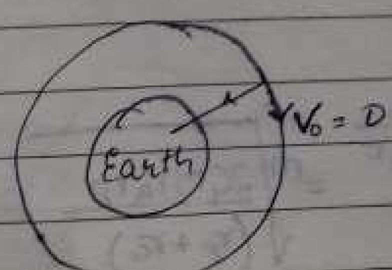
$$v_1 + v_2 = \sqrt{\frac{2G(M+m)^{\cancel{2}}}{\cancel{(m+M)}d}}$$

$$\Rightarrow \sqrt{\frac{2G(M+m)}{d}} \quad \text{Ans}$$

Q A satellite is revolving round the earth with orbital speed v_0. If it stops suddenly, the speed with which it will strike the surface of earth would be

(v_e = escape velocity of a particle on earth's surface)

Ans



By Conservation of Energy

$$\Rightarrow -\frac{GMm}{R+h} = \frac{1}{2}mv_x^2 - \frac{GMm}{R}$$

$$\Rightarrow \frac{GM\cancel{m}}{R} - \frac{GM\cancel{m}}{R+h} = \frac{1}{2}\cancel{m}v_x^2$$

LIMITED EDITION

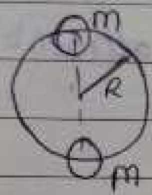
$$\frac{2GM}{R} - \frac{2GM}{R+h} = v_x^2$$

$$\Rightarrow v_e^2 - 2v_o^2 = v_x^2$$

$$v_x = \sqrt{v_e^2 - 2v_o^2} \quad \underline{\text{Ans}}$$

Q Two particles each of mass M, move along a circle of radius R under the action of their mutual gravitational attraction. The speed of each particle is.

Ans



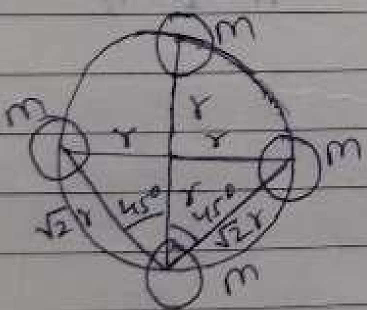
$F_c = F_g$

$$\Rightarrow \frac{mv^2}{R} = \frac{Gm^2}{4R^2}$$

$$v = \sqrt{\frac{Gm}{4R}} = \sqrt{\frac{Gm}{4R}} \quad \underline{\text{Ans}}$$

Q ~~two~~ Four particles each of mass M, move along a circle of Radius R under the action of their mutual gravitational attraction. The speed of each particle is.

Ans



Net force experienced by each mass

$$\Rightarrow \frac{Gm^2}{4R^2} + \frac{\sqrt{2}\,Gm^2}{2R^2} = \frac{mv^2}{R}$$

$$\Rightarrow \frac{Gm^2}{2R^2}\left(\frac{1}{2} + \sqrt{2}\right) = \frac{mv^2}{R}$$

$$\Rightarrow \frac{Gm}{2R}\left(\frac{1+2\sqrt{2}}{2}\right) = v^2$$

$$\Rightarrow \sqrt{\frac{Gm}{R}\left(\frac{1+2\sqrt{2}}{4}\right)} = v \quad \underline{\text{Ans}}$$

Q Two particles of equal mass are moving round a circle of radius r due to their mutual attraction interaction. Find the Time period of each particle.

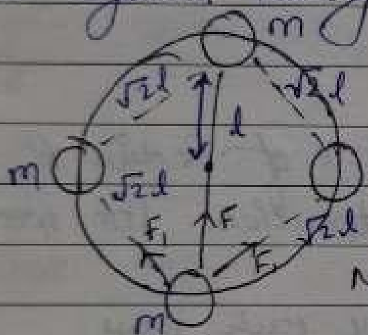
Ans
$$\frac{\cancel{m}v_0^2}{\cancel{R}} = \frac{Gm^{\cancel{2}}}{4R^{\cancel{2}}}$$

$$\Rightarrow v_0 = \sqrt{\frac{Gm}{4R}}$$

$$T = \frac{2\pi r}{v}$$

Q Four particles of equal mass are moving round a circle of radius r due to their mutual gravitational attraction. Find the angular velocity of each particle.

Ans



Net Force (centripetal) $= \vec{F_1} + \vec{F_1} + \vec{F}$

$$\Rightarrow F_1 = \frac{Gm^2}{2l^2} \qquad \vec{F_1} + \vec{F_2} = \vec{F_3}$$

$$\vec{F} = \frac{Gm^2}{4l^2}$$

$\vec{F_3}$

$$\text{Net force} = \sqrt{2}\frac{Gm^2}{2l^2} + \frac{Gm^2}{4l^2}$$

$$m\omega^2 r = \sqrt{2}\frac{Gm^2}{2l^2} + \frac{Gm^2}{4l^2}$$

$$\cancel{m}\omega^2 l = \frac{Gm^{\cancel{2}}}{2l^2}\left(\sqrt{2} + \frac{1}{42}\right)$$

$$\omega = \sqrt{\frac{Gm}{l^3}\left(\frac{2\sqrt{2}+1}{4}\right)} \quad \text{Ans}$$

Q If radius of earth contracted by 0.1%, its mass remains same, then wt. of the body at ~~radius~~ earth's surface will increase by.

Ans $g = \frac{GM}{R^2}$

$$100 \times \frac{\Delta g}{g} = -2\left(\frac{\Delta R}{R}\right) \times 100$$

$\Rightarrow +.2\%$

Q If ~~mass~~ radius of earth increases by 50% and mass of earth decreases by 25%, then accn due to gravity at its surface decreases by nearly.

$M' = 75\% \, M = \frac{3}{4}M$

$R' = 150\% \, R = \frac{3R}{2}$

$$g = \frac{GM}{R^2} = \frac{3GM \times 4}{4 \times 9R^2} = \frac{GM}{3R^2} = 0.33\frac{GM}{3R^2}$$ decreases by 67%

Q If masses of two point objects are tripled and distance b/w them is doubled, then gravitational force of attraction b/w them will.

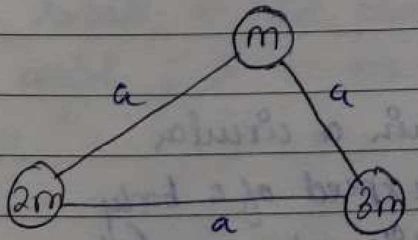
Ans $m_1' = 3m_1$

$m_2' = 3m_2$ $\quad F' = \frac{9Gm_1m_2}{4R^2} = 2.25 \times 100 = 225\%$

$r' = 2r$ $\quad$ Increases by 125% Ans

Q Three particles of masses m, 2m, and 3m are placed at the corners of an equilateral triangle of side a. Calculate.

(i) The P.E. of the system



$$U = -\frac{2Gm^2}{a} - \frac{6Gm^2}{a} - \frac{3m^2}{a} \Rightarrow -\frac{11Gm^2}{a}$$ Ans

(ii) The work done on the system if the side of the triangle is changed from a to 2a. Assume the p.E. to be zero when the separation is infinity.

Ans $W = \Delta U$

$\Rightarrow U_f - U_i$

$\Rightarrow -\frac{11Gm^2}{2a} + \frac{11Gm^2}{a} = \frac{11Gm^2}{2a}$

Q Calculate the energy required to put a satellite of mass m from earth surface into a orbit of radius, $4R_E$ where R_E is radius of earth

Ans $\Delta E = T.E_f - T.E_i$

Potential + Kinetic energy

Potential energy

$\Rightarrow -\frac{GMm}{8R_e} + \frac{GMm}{R}$

$\Rightarrow \frac{GMm}{R}\left(1 - \frac{1}{8}\right)$

$\Rightarrow \frac{7}{8}\frac{GMm}{R}$ Ans

Q Let escape velocity of a body kept at surface of a planet is u. If it is projected at a speed of 200% more than the escape speed, then its escape speed in interstellar space will be

Ans $u' = 300\%$ of u

$u' = 3u$

$\sqrt{v_p^2 - v_e^2} = v_s$

$\Rightarrow \sqrt{9v_e^2 - v_e^2} = v_s$

$\Rightarrow \sqrt{8u^2} = v_s$

$\Rightarrow 2\sqrt{2}u = v_s$ Ans

Q An artificial satellite is moving around earth in a circular orbit with speed equal to one fourth the escape speed of a body from the surface of earth. The height of satellite above earth's surface is.

Ans $v_e = \sqrt{\frac{2GM}{R}}$

$\frac{1}{4}v_e = v_0 = \sqrt{\frac{GM}{R+h}}$

$\frac{1}{8}\frac{\cancel{2}GM}{\cancel{16}\,R} = \frac{GM}{R+h}$

$\Rightarrow R+h = 8R$

$h = 7R$

Q A satellite of mass 200 kg revolves around a planet of mass 5×10^{30} kg in a circular orbit of radius 6.6×10^{6} m. Binding energy of the satellite is.

Ans $B.E. = |T.E.| = \frac{GMm}{2r} = \frac{\cancel{6.6\times10^{-11}} \times 5\times10^{\cancel{30}\,13} \times \cancel{200}\,100}{\cancel{2\times6.6\times10^{6}}} \Rightarrow 5\times10^{15}$ J Ans

Q Two satellites A and B go round the planet P in circular orbits having radii 4R and R respectively. If the speed of the satellite A is 3v. the speed of satellite B will be.

Ans $v = \sqrt{\frac{GM}{R}}$

$\frac{v_1}{v_2} = \frac{\sqrt{R_2}}{\sqrt{R_1}} \Rightarrow \frac{3v}{v_2} = \frac{\sqrt{R}}{\sqrt{4R}} \Rightarrow \frac{9v^2}{v_2^2} = \frac{R}{4R} \Rightarrow v_2 = 6v$ Ans

Q Mark the correct statements.

(i) Escape velocity does not depend on mass of the body.

(ii) If total energy of the satellite becomes +ve, it escapes from earth.

(iii) Orbit of geostationary satellite is called parking orbit.

Ans (i), (ii), (iii) all are correct.

Q What should be the angular speed with which the earth have to rotate on its axis so that a person upon the equator would weigh $\frac{3}{5}$ th as much as present?

Ans $g_{eff} = g - \omega^2 R$

$\Rightarrow \frac{3}{5}g = g - \omega^2 R$

$\Rightarrow \frac{2g}{5} = \omega^2 R$

$\omega = \sqrt{\frac{2g}{5R}}$ Ans

LIMITED EDITION

Q If potential energy of a body of mass m on the surface of earth is taken as zero then its P.E. at height h above the surface of earth is.

Ans $\Delta U = \frac{mgh}{1+\frac{h}{R}}$

$U_f - \cancel{U_i}^{0} = \frac{mgh}{1+\frac{h}{R}}$

$U_h = \frac{m\,GM\,h}{R^2\frac{(R+h)}{R}} = \frac{GMmh}{R(R+h)}$ Ans

Q Q A particle is projected vertically up with velocity $v = \sqrt{\frac{4gR}{3}}$ from earth surface. The velocity of particle at height equal to half of the max^m height reached by it.

Ans $h = \frac{R}{\frac{2gR}{2\frac{4gR}{3}}-1} = \frac{R}{\frac{3}{2}-1} = 2R$

By C.O.M.E.

$-\frac{GM\cancel{m}}{R} + \frac{1}{\cancel{2}}\cancel{m}\frac{4}{3}\frac{GM}{R} = \frac{-GM\cancel{m}}{2R} + \frac{1}{\cancel{2}}\cancel{m}v^2$

$\Rightarrow \frac{GM}{R} - \frac{2GM}{R} + \frac{4}{3}\frac{GM}{R} = v^2 + \frac{GM}{R}$

$\Rightarrow -\frac{GM}{R} + \frac{4GM}{3R} = v^2$

$\Rightarrow \frac{GM}{3R} = v^2 \Rightarrow \frac{gR^{\cancel{2}}}{3\cancel{R}} = v^2$

$v = \sqrt{\frac{gR}{3}}$

Q When energy of a satellite-planet system is +ve then satellite will.

Ans Escape out with speed greater than escape velocity.

Q If L is the angular momentum of a satellite revolving around earth in a circular orbit of radius r with speed v, then.

a) $L \propto v$ (b) $L \propto r$

(c) $L \propto \sqrt{r}$ (d) $L \propto \sqrt{v}$

Ans $L = mvr \Rightarrow m\sqrt{\frac{GM}{r}}\, r$

$$\Rightarrow m\sqrt{\frac{GM r^2}{r}} = m\sqrt{GMr} \quad \therefore L \propto \sqrt{r} \text{ Ans}$$

Q The time period of a geostationary satellite is 24h, at a height $6R_e$ from surface of earth. The time period of another satellite whose height is $2.5R_e$ from surface will be.

Ans $\frac{T_1^2}{T_2^2} = \frac{R_1^3}{R_2^3}$ $\quad R_1 = 6R_e + 1R_e = 7R_e$

$R_2 = 2.5R_e + R_e = 3.5R_e$

$$\Rightarrow \frac{576}{T_2^2} = \left(\frac{\cancel{7}^{2} \cancel{R_e}}{\cancel{3.5} \cancel{R_e}}\right)^3$$

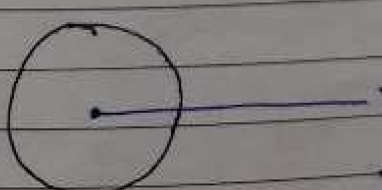
$$\Rightarrow \frac{576}{T_2^2} = 8$$

$$T_2^2 = \cancel{\frac{576}{8}}\ 72$$

$$T_2 = \sqrt{72}$$

$$T_2 = 6\sqrt{2} = 6\sqrt{2} \text{ Ans}$$

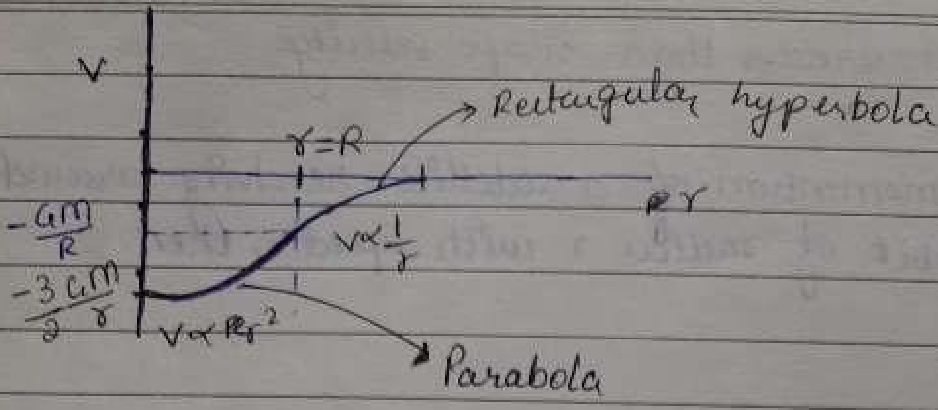
GRAPH FOR THE POTENTIAL OF SOLID SPHERE WITH r



$V_{out} = \frac{-GM}{r} \; (r > R)$ $\quad V_{inside} = +\frac{GM}{2R^3}(r^2 - 3R^2)$

$V \propto \frac{1}{r}$

$V_{surface} = \frac{-GM}{R} \; (r = R)$



Q The minimum and maximum distance of a satellite from the centre of earth is 2R and 4R respectively. Its maximum speed is

Ans ~~$m_1 v_1 = m_2 v_2$~~

or $r_1 v_1 = r_2 v_2$

$\cancel{2R} \times v_1 = \cancel{4R}^{2} v_2$

$v_1 = 2v_2$

Now apply law of conservation of ~~momentum~~ Energy

$$-\frac{GMm}{2R} + \frac{1}{2}mv_1^2 = -\frac{GMm}{4R} + \frac{1}{2}m\left(\frac{v_1}{2}\right)^2$$

$$\Rightarrow -\frac{GM\cancel{m}}{2\cancel{4}R} + \frac{1}{\cancel{2}}\cancel{m}v_1^2 = \frac{1}{\cancel{2}}\cancel{m}\frac{v_1^2}{4}$$

$$\Rightarrow v_1^2 = \frac{v_1^2}{4} + \frac{GM}{2R}$$

$$\Rightarrow \frac{3v_1^2}{\cancel{4}_2} = \frac{GM}{\cancel{2}R}$$

$$\Rightarrow v_1 = \sqrt{\frac{2GM}{3R}} \quad \text{Ans}$$

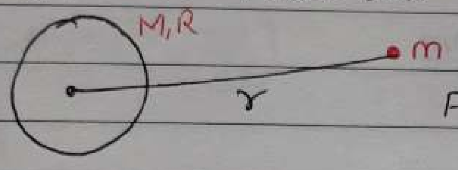
FORMULA SHEET

1. Magnitude of gravitational force b/w two point objects $= \frac{Gm_1m_2}{r^2}$

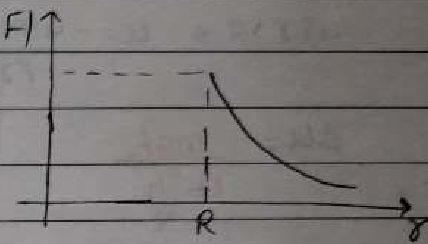
2. Vector form of gravitational force $= -\frac{Gm_1m_2}{r^2}\hat{r}_{21}$

3. A point from small mass (m) where the gravitational force/Field is zero $= \frac{d}{\sqrt{n}+1}$ $\quad n = \frac{M}{m}$

4. GRAVITATIONAL FORCE DUE TO HOLLOW SPHERE



$F = \frac{GMm}{r^2}$

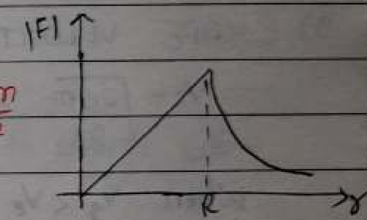


ON THE SURFACE $= \frac{GMm}{R^2}$ $\quad$ INSIDE $= 0$

5) GRAVITATION FORCE DUE TO SOLID SPHERE

⇒ Outside $(R<r) \Rightarrow -\frac{GMm}{r^2}$ $\quad$ On the surface $(R=r)$ $\frac{GMm}{R^2}$

Inside $= -\frac{GMmr}{R^3}$



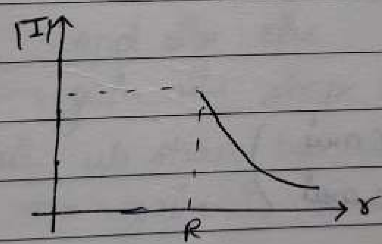
6) GRAVITATIONAL FIELD INTENSITY

1) HOLLOW SPHERE

a) $(r>R) = \frac{GM}{r^2}$

b) $(R=r) = \frac{GM}{R^2}$

c) $(R>r) = 0$

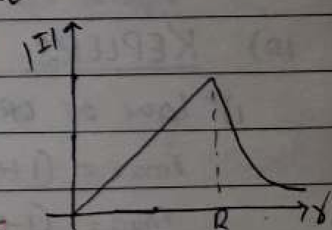


SOLID SPHERE

a) $r>R = \frac{GM}{r^2}$

b) $r=R = \frac{GM}{R^2}$

c) $r<R = \frac{GMr}{R^3}$



7) ACCELERATION DUE TO GRAVITY

Case 1 → 0 → 20 km

$\Rightarrow g = g$

Case 2 → 20 → 500 km

$g = g\left(1 - \frac{2h}{R}\right)$

Case – 3 → >500

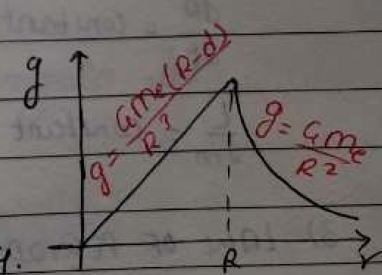
$g_{eff} = \frac{gR^2}{(R+h)^2}$

Case 4 (Inside earth)

$g_{eff} = g\left(1 - \frac{d}{R}\right)$

% change in acc^n due to gravity.

$\frac{\Delta g}{g} \times 100 = -\frac{2h}{R} \times 100$



8) GRAVITATIONAL POTENTIAL ENERGY

$$U = -\frac{GMm}{r}$$

$$I = -\frac{dU}{dr} \Rightarrow -\left(\frac{\partial U}{dx}\hat{i} + \frac{\partial U}{dy}\hat{j} + \frac{\partial U}{dz}\hat{k}\right)$$

SOLID SPHERE

1) $r > R \Rightarrow U = -\frac{GMm}{r}$ 2) $r = R = -\frac{GMm}{R}$

HOLLOW SPHERE

1) $r > R \Rightarrow U = -\frac{GMm}{Pr}$ 2) $r = R \Rightarrow -\frac{GMm}{R}$

$$\Delta U = \frac{mgh}{1 + \frac{h}{R}}$$

9) ESCAPE VELOCITY

$$V_e = \sqrt{\frac{2GM}{R}} \Rightarrow \sqrt{\frac{8}{3}\pi \rho G R^2}$$

When $v_g < V_e$ $h = \frac{R_e}{\frac{V_e^2}{v_g^2} - 1} = \frac{V^2}{2g - \frac{V^2}{R}}$

10) KEPLERS LAW

1) LAW OF ORBITS

$r_{max} = (1+e)a$ (a = Semi major axis

$r_{min} = (1-e)a$ b = semi minor axis)

2) LAW OF AREA

$r\vec{v} =$ constant

$\frac{dA}{dt} =$ constant

$\frac{L}{2m} =$ constant

3) LAW OF PERIOD

$T^2 = a^3$